1. Three students are working on a problem involving a differentiable function $f(x, y, z)$ at a point P_0.

(1) *Student A* says: An equation for the tangent plane to the level surface of f passing through P_0 is $2x + 3y - 6z = 11$.

(2) *Student B* says: The largest possible rate of change of f at P_0 in any direction is 25.

(3) *Student C* says: The directional derivative of f at P_0 in the direction of the vector $\mathbf{A} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ is 5.

Student D, who has been listening in, but does not know anything about the function f, decides that at least one of the *Students A*, *B*, *C* must have made a mistake in their calculations.

Explain *Student D*'s reasoning.

(1) $\Rightarrow$ $\vec{\nabla} f(P_0) = c \cdot (2\vec{i} + 3\vec{j} - 6\vec{k})$ for some scalar c

(2) $\Rightarrow$ $|\vec{\nabla} f(P_0)| = 25$

(3) $\Rightarrow$ $D_{\vec{u}} f(P_0) = 5$ where $\vec{u} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{2\vec{i} + 2\vec{j} + \vec{k}}{3} \Rightarrow \vec{\nabla} f(P_0) \cdot \vec{u} = 5$

(1) and (2) $\Rightarrow$ $c^2 \cdot (2^2 + 3^2 + (-6)^2) = 25^2 \Rightarrow c^2 = \dfrac{25^2}{49} \Rightarrow c = \pm \dfrac{25}{7}$

(1) and (3) $\Rightarrow$ $c \cdot (2\vec{i} + 3\vec{j} - 6\vec{k}) \cdot \dfrac{2\vec{i} + 2\vec{j} + \vec{k}}{3} = 5 \Rightarrow \dfrac{4}{3} c = 5 \Rightarrow c = \dfrac{15}{4}$

As $\dfrac{25}{7} \neq \dfrac{15}{4}$ and $-\dfrac{25}{7} \neq \dfrac{15}{4}$, (1), (2), (3) cannot be all true statements.