

3. Evaluate the following integrals.

$$\text{a. } \int (3x-1)(x+2)(x^2-2)^{2022}(2x+5)^{2022} dx \equiv \int u^{2022} \cdot \frac{1}{2} du$$

$$u = (x^2-2)(2x+5) = 2x^3 + 5x^2 - 4x - 10$$

$$du = (6x^2 + 10x - 4) dx = 2(3x-1)(x+2) dx$$

$$= \frac{1}{2} \cdot \frac{u^{2023}}{2023} + C' = \frac{(x^2-2)^{2023} (2x+5)^{2023}}{4046} + C'$$

$$\text{b. } \int_{\pi/4}^{5\pi/4} \frac{\cos^2 x}{(x + \sin x \cos x)^2} dx \equiv \int_{\frac{\pi}{4} + \frac{1}{2}}^{\frac{5\pi}{4} + \frac{1}{2}} \frac{1}{u^2} \cdot \frac{1}{2} du = \frac{1}{2} \int_{\frac{\pi}{4} + \frac{1}{2}}^{\frac{5\pi}{4} + \frac{1}{2}} u^{-2} du$$

$$u = x + \sin x \cos x$$

$$du = (1 + \cos^2 x - \sin^2 x) dx = 2 \cos^2 x dx$$

$$= -\frac{1}{2} u^{-1} \Big|_{\frac{\pi}{4} + \frac{1}{2}}^{\frac{5\pi}{4} + \frac{1}{2}} = -\frac{1}{2} \cdot \left(\frac{1}{\frac{5\pi}{4} + \frac{1}{2}} - \frac{1}{\frac{\pi}{4} + \frac{1}{2}} \right) = \frac{8\pi}{(5\pi+2)(\pi+2)}$$