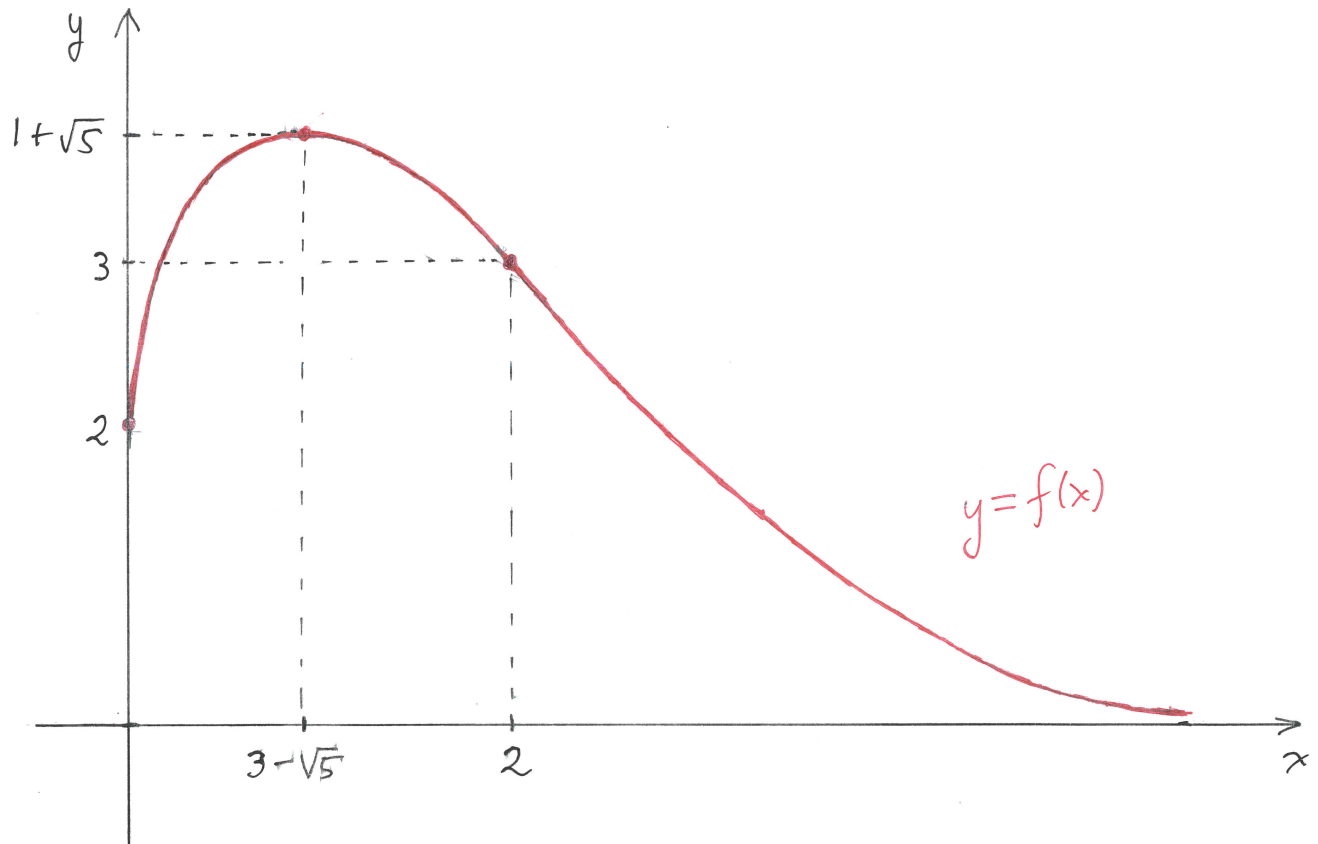


4. A function f , which is continuous on $[0, \infty)$ and twice-differentiable on $(0, \infty)$, satisfies the following conditions:

- ① $f(0) = 2$, $f(3 - \sqrt{5}) = 1 + \sqrt{5}$, $f(2) = 3$
- ② $\lim_{x \rightarrow \infty} f(x) = 0$
- ③ $f'(x) > 0$ for $0 < x < 3 - \sqrt{5}$, and $f'(x) < 0$ for $x > 3 - \sqrt{5}$
- ④ $\lim_{x \rightarrow 0^+} f'(x) = \infty$
- ⑤ $f''(x) < 0$ for $0 < x < 2$, and $f''(x) > 0$ for $x > 2$

a. Sketch the graph of $y = f(x)$ making sure that all important features are clearly shown.



b. Fill in the boxes to make the following a true statement. No explanation is required.

The function $f(x) = \frac{a\sqrt{x} + b}{x + c}$ satisfies the conditions ①-⑤ at all points in its domain if a , b and c are chosen as

$$a = \boxed{4\sqrt{2}}, \quad b = \boxed{4} \quad \text{and} \quad c = \boxed{2}.$$