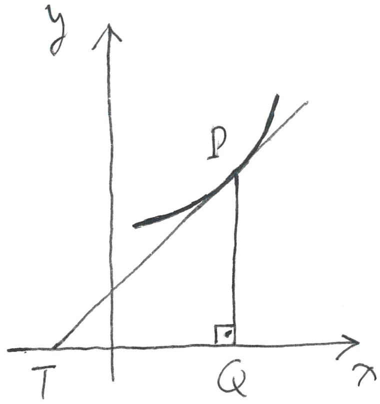
3. For a point P on the curve $y = x^2 + 5$ different from $(0, 5)$, let T and Q be points on the x-axis such that the line PT is tangent to the curve at P and PQ is perpendicular to the x-axis. Let A be the area of the triangle PQT. Assume that the coordinates are measured in centimeters and the time is measured in seconds.

Find all possible values of the x-coordinate of the point P at a moment when A is decreasing at a rate of 6 cm^2/s and the x-coordinate of the point P is increasing at a rate of 2 cm/s.



$$\tan(\widehat{PTQ}) = \frac{|PQ|}{|TQ|} \Rightarrow |y'| = \frac{|y|}{|TQ|} \Rightarrow |TQ| = \frac{|y|}{|y'|}$$

$$A = \frac{1}{2}\cdot|TQ|\cdot|PQ| = \frac{1}{2}\cdot\frac{|y|}{|y'|}\cdot|y| = \frac{y^2}{2|y'|} = \frac{(x^2+5)^2}{2\cdot|2x|} = \frac{(x^2+5)^2}{4|x|}$$

$$\Rightarrow A = \begin{cases} \frac{1}{4}\left(x^3+10x+\frac{25}{x}\right) & \text{if } x>0 \\ -\frac{1}{4}\left(x^3+10x+\frac{25}{x}\right) & \text{if } x<0 \end{cases} \Rightarrow \frac{dA}{dx} = \begin{cases} \frac{1}{4}\left(3x^2+10-\frac{25}{x^2}\right) & \text{if } x>0 \\ -\frac{1}{4}\left(3x^2+10-\frac{25}{x^2}\right) & \text{if } x<0 \end{cases}$$

$$\frac{dA}{dt} = \frac{dA}{dx}\cdot\frac{dx}{dt} \xRightarrow{\text{at our moment}} -6 = \frac{dA}{dx}\cdot 2 \Rightarrow \frac{dA}{dx} = -3$$

Therefore:

If $x>0$, then $\frac{1}{4}\left(3x^2+10-\frac{25}{x^2}\right) = -3 \Rightarrow \underbrace{3x^4+22x^2-25}_{(x^2-1)(3x^2+25)}=0 \Rightarrow x=1$

If $x<0$, then $-\frac{1}{4}\left(3x^2+10-\frac{25}{x^2}\right) = -3 \Rightarrow 3x^4-2x^2-25=0 \Rightarrow x^2 = \frac{2\pm\sqrt{304}}{6}$

$$\Downarrow$$

$$x = -\sqrt{\frac{1+2\sqrt{19}}{3}}$$

The possible values of x are 1 and $-\sqrt{\frac{1+2\sqrt{19}}{3}}$.