4. In each of the following, if there exists a function f that satisfies the given conditions, give an example of such a function; otherwise, just write DOES NOT EXIST inside the box. No explanation is required. No partial points will be given.

a. f is continuous on $(-\infty, \infty)$ and f does not have an antiderivative on $(-\infty, \infty)$.

$f(x) =$ DNE

b. f is positive and differentiable on $(-\infty, \infty)$ and $\displaystyle\int \frac{dx}{f(x)} \neq \ln(f(x)) + C$.

$f(x) =$ x^2+1

c. f is continuous on $[0, \pi]$ and $\displaystyle\int_0^{\pi} |f(x)|\, dx \neq \left| \int_0^{\pi} f(x)\, dx \right|$.

$f(x) =$ $\cos x$

d. f is differentiable on $(0, \infty)$, $\lim_{x\to\infty} f'(x) = 0$, and $\lim_{x\to\infty} f(x)$ does not exist.

$f(x) =$ $\sqrt{x}$

e. f is differentiable on $(0, \infty)$, $\lim_{x\to\infty} f(x) = 0$, and $\lim_{x\to\infty} f'(x)$ does not exist.

$f(x) =$ $\dfrac{\sin(x^2)}{x}$