2a. Evaluate the improper integral $\int_0^\infty \frac{dx}{(x+1)(2x+3)}$.

$$\int \frac{dx}{(x+1)(2x+3)} = \int \left(\frac{1}{x+1} - \frac{2}{2x+3}\right) dx = \ln|x+1| - \ln|2x+3| + C$$

$$\int_0^\infty \frac{dx}{(x+1)(2x+3)} = \lim_{c\to\infty} \int_0^c \frac{dx}{(x+1)(2x+3)} = \lim_{c\to\infty} \left[\ln\left|\frac{x+1}{2x+3}\right|\right]_0^c$$

$$= \lim_{c\to\infty} \left(\ln\left(\frac{c+1}{2c+3}\right) - \ln\left(\frac{1}{3}\right)\right) = \ln\left(\frac{1}{2}\right) - \ln\left(\frac{1}{3}\right) = \ln\left(\frac{3}{2}\right)$$

2b. Let $A = \int_1^{\sqrt{3}} \frac{\ln x}{x^2+1}\, dx$ and $B = \int_2^{2\sqrt{3}} \frac{\ln x}{x^2+4}\, dx$. Express B in terms of A.

$$B = \int_2^{2\sqrt{3}} \frac{\ln x}{x^2+4}\, dx = \int_1^{\sqrt{3}} \frac{\ln(2u)}{4u^2+4} \cdot 2\, du = \frac{1}{2} \int_1^{\sqrt{3}} \frac{\ln 2 + \ln u}{u^2+1}\, du$$

$$\boxed{\begin{array}{c} x = 2u \\ dx = 2du \end{array}}$$

$$= \frac{\ln 2}{2} \int_1^{\sqrt{3}} \frac{du}{u^2+1} + \frac{A}{2} = \frac{\ln 2}{2} \Big[\arctan u\Big]_1^{\sqrt{3}} + \frac{A}{2}$$

$$= \frac{\ln 2}{2} \cdot \big(\underbrace{\arctan\sqrt{3}}_{\frac{\pi}{3}} - \underbrace{\arctan 1}_{\frac{\pi}{4}}\big) + \frac{A}{2} = \frac{\pi \ln 2}{24} + \frac{A}{2}$$