1. Evaluate the following limits. [Do not use L'Hôpital's Rule!]

a. $\displaystyle\lim_{x\to-\infty} \frac{\sin(1/x)}{\sqrt{x^2+1}+x} = \lim_{x\to-\infty} \frac{\sin(1/x)\cdot(\sqrt{x^2+1}-x)}{(\sqrt{x^2+1}+x)(\sqrt{x^2+1}-x)}$

$$= \lim_{x\to-\infty} \left(\sin(1/x)\cdot\left(|x|\cdot\sqrt{1+\frac{1}{x^2}}-x\right)\right)$$

$$= \lim_{x\to-\infty} \left(\sin(1/x)\cdot\left(-x\sqrt{1+\frac{1}{x^2}}-x\right)\right)$$

$$= -\underbrace{\lim_{x\to-\infty} \frac{\sin(1/x)}{1/x}}_{1} \cdot \lim_{x\to-\infty}\left(\sqrt{1+\frac{1}{x^2}}+1\right) = -1\cdot(1+1) = -2$$

b. $\displaystyle\lim_{x\to\pi/2} \frac{1+\cos 2x}{1-\sqrt[3]{\sin x}} = \lim_{x\to\frac{\pi}{2}} \frac{2\cos^2 x\cdot(1+(\sin x)^{1/3}+(\sin x)^{2/3})}{(1-(\sin x)^{1/3})\cdot(1+(\sin x)^{1/3}+(\sin x)^{2/3})}$

$$= 2\lim_{x\to\frac{\pi}{2}} \frac{1-\sin^2 x}{1-\sin x}\cdot\lim_{x\to\frac{\pi}{2}}(1+(\sin x)^{1/3}+(\sin x)^{2/3})$$

$$= 2\cdot\lim_{x\to\frac{\pi}{2}}(1+\sin x)\cdot(1+1+1) = 2\cdot(1+1)\cdot 3 = 12$$