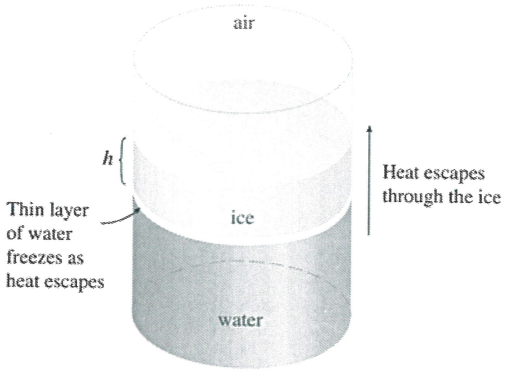
5. The thickness h of a sheet of sea ice as a function of time t satisfies the equation

$$\frac{dh}{dt} = \frac{k}{LDh}(T_w - T_a)$$

for $h > 0$ where

- T_a is the air temperature,
- T_w is the water temperature,
- k is the thermal conductivity of the ice,
- L is the latent heat of the water, and
- D is the mass density of the water.



In this question we will assume that T_a, T_w, k, L, and D are constants.

We measure the thickness of a sheet of sea ice daily, and observe that it is 3 cm on day 0, and 5 cm on day 10.

Determine when the thickness of the sheet will be 13 cm.

Let $A = \frac{k}{LD} \cdot (T_w - T_a)$.

$\frac{dh}{dt} = \frac{A}{h} \Rightarrow h\,dh = A\,dt \Rightarrow \int h\,dh = \int A\,dt$

$\Rightarrow \frac{1}{2}h^2 = At + C'$

$t=0$, $h=3$ and $t=10$, $h=5$

$\frac{9}{2} = C$ and $\frac{25}{2} = 10A + C' \Rightarrow A = \frac{4}{5}$

$\frac{1}{2}h^2 = \frac{4}{5}t + \frac{9}{2}$

$h = 13$

$\frac{169}{2} = \frac{4}{5}t + \frac{9}{2} \Rightarrow t = 100$ days

The thickness of the sheet will be 13 cm on day 100.