4a. Find $f(8)$ if f is a contiuous function satisfying

$$\int_0^{x^3} f(t)\,dt + \int_0^{x} f(t^3)\,dt = x$$

for all x.

$$\Big\Downarrow \, d/dx$$

$$\frac{d}{dx}\int_0^{x^3} f(t)\,dt + \frac{d}{dx}\int_0^{x} f(t^3)\,dt = \frac{d}{dx}x$$

$$\Big\Downarrow \leftarrow \boxed{\text{FTC1}}$$

$$f(x^3)\cdot 3x^2 + f(x^3) = 1$$

$$\Big\Downarrow \leftarrow \boxed{x=2}$$

$$f(8)\cdot 12 + f(8) = 1$$

$$\Big\Downarrow$$

$$f(8) = \frac{1}{13}$$

4b. Evaluate the limit $\lim\limits_{x\to 0} \dfrac{e^{x-\frac{1}{2}x^2} - x - 1}{x^3}$.

$$\lim_{x\to 0} \frac{e^{x-\frac{1}{2}x^2} - x - 1}{x^3} \overset{\text{l'H}}{=} \lim_{x\to 0} \frac{e^{x-\frac{1}{2}x^2}\cdot(1-x) - 1}{3x^2} \overset{\text{l'H}}{=} \lim_{x\to 0} \frac{e^{x-\frac{1}{2}x^2}\cdot(1-x)^2 - e^{x-\frac{1}{2}x^2}}{6x}$$

$$= \lim_{x\to 0}\left(\frac{1}{6}\cdot e^{x-\frac{1}{2}x^2}\cdot\frac{1-2x+x^2-1}{x}\right) = \frac{1}{6}\cdot e^0\cdot\lim_{x\to 0}(-2+x) = \frac{1}{6}\cdot 1\cdot(-2) = -\frac{1}{3}$$