2. Evaluate the following integrals.

a. $\int \frac{\sin^3 x}{1+\cos x}\,dx = \int \frac{\sin^2 x}{1+\cos x} \sin x \, dx = \int \frac{1-\cos^2 x}{1+\cos x} \sin x \, dx$

$= \int (1-\cos x)\cdot \sin x \, dx = \int (1-u)\cdot(-du) = -u + \frac{1}{2}u^2 + C$

$\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array}}$

$= -\cos x + \frac{1}{2}\cos^2 x + C$

b. $\int_0^{\pi/2} \frac{\sin^3 x}{1+\cos^2 x}\,dx = \int_0^{\pi/2} \frac{\sin^2 x}{1+\cos^2 x}\cdot \sin x \, dx = \int_0^{\pi/2} \frac{1-\cos^2 x}{1+\cos^2 x}\cdot \sin x \, dx$

$= \int_1^0 \frac{1-u^2}{1+u^2}\cdot(-du) = \int_0^1 \left(\frac{2}{1+u^2} - 1\right) du = \Big[2\arctan u - u\Big]_0^1$

$\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array}}$

$= 2\arctan(1) - 1 - 2\arctan(0) + 0$

$= 2\cdot\frac{\pi}{4} - 1 = \frac{\pi}{2} - 1$