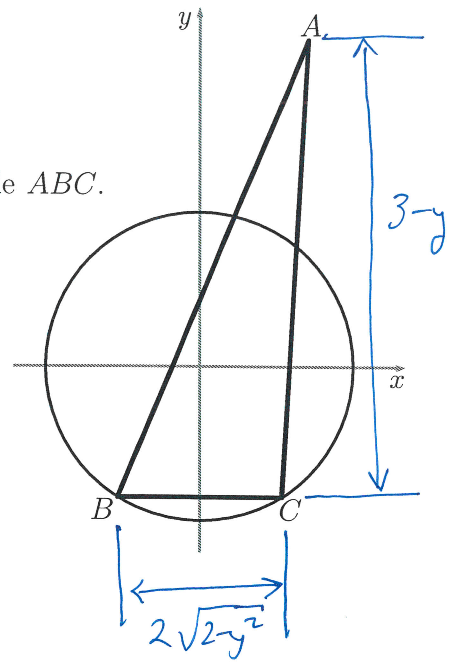
1. A triangle ABC in the xy-plane satisfies the following conditions:

❶ The coordinates of the vertex A are $(1, 3)$.

❷ The vertices B and C lie on the circle $x^2 + y^2 = 2$.

❸ The side $[BC]$ is parallel to the x-axis.

Find the largest and smallest possible values of the area of the triangle ABC.



Let y be the y-coordinate of B.

Then:

$$S = (\text{Area of } ABC) = \frac{1}{2} \cdot 2\sqrt{2-y^2} \cdot (3-y)$$

We want to:

Maximize/Minimize $S = (3-y) \cdot \sqrt{2-y^2}$ for $-\sqrt{2} \leq y \leq \sqrt{2}$

Critical points:

$$\frac{dS}{dy} = -\sqrt{2-y^2} + (3-y) \cdot \frac{-2y}{2\sqrt{2-y^2}} = 0 \Rightarrow 2y^2 - 3y - 2 = 0$$

$$\Rightarrow y = \frac{3 \pm 5}{4} = \begin{cases} 2 \;\otimes \leftarrow \text{not in the interval} \\ -1/2 \end{cases}$$

$$y = -\frac{1}{2} \Rightarrow S = \frac{7\sqrt{7}}{4}$$

End points:

$y = \sqrt{2} \Rightarrow S' = 0$

$y = -\sqrt{2} \Rightarrow S = 0$

The largest possible area is $\frac{7\sqrt{7}}{4}$.

The smallest possible area is 0.