4. Find the absolute maximum and minimum values of $f(x) = -x^4 + \frac{21}{2}x^2 - 10x$ on the interval $[-3, 3]$.

$$f'(x) = -4x^3 + 21x - 10 \underset{\uparrow}{=} -(x-2)\underbrace{(4x^2+8x-5)}_{(2x-1)(2x+5)} = 0 \Rightarrow x = 2,\ x = \frac{1}{2},\ x = -\frac{5}{2}$$

$\boxed{f'(2) = 0}$

Critical points:

$x = 2 \Rightarrow f(2) = -2^4 + \frac{21}{2} \cdot 2^2 - 10 \cdot 2 = 6$

$x = \frac{1}{2} \Rightarrow f(\frac{1}{2}) = -(\frac{1}{2})^4 + \frac{21}{2} \cdot (\frac{1}{2})^2 - 10 \cdot \frac{1}{2} = -\frac{39}{16}$

$x = -\frac{5}{2} \Rightarrow f(-\frac{5}{2}) = -(-\frac{5}{2})^4 + \frac{21}{2} \cdot (-\frac{5}{2})^2 - 10 \cdot (-\frac{5}{2}) = \frac{825}{16}$ $\leftarrow$ largest

Endpoints:

$x = -3 \Rightarrow f(-3) = -(-3)^4 + \frac{21}{2} \cdot (-3)^2 - 10 \cdot (-3) = \frac{87}{2}$

$x = 3 \Rightarrow f(3) = -3^4 + \frac{21}{2} \cdot 3^2 - 10 \cdot 3 = -\frac{33}{2}$ $\leftarrow$ smallest

Absolute maximum is $\frac{825}{16}$.

Absolute minimum is $-\frac{33}{2}$.