

Do not forget to write your full name and your Bilkent ID number, and sign on the upper right corner of your paper.

Final Exam Question 2.

Evaluate the limit $\lim_{x \rightarrow 0} \frac{x^2 \cos x - x \sin x}{\ln(1+x^4)}$.

Show all your work!

Explain your reasoning fully and in detail using correct mathematical notation and terminology, and in well-formed mathematical and English sentences!

$$\lim_{x \rightarrow 0} \frac{x^2 \cos x - x \sin x}{\ln(1+x^4)} = \lim_{x \rightarrow 0} \frac{x^2 \cos x - x \sin x}{x^4} \cdot \lim_{x \rightarrow 0} \frac{x^4}{\ln(1+x^4)}$$

$$= \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x^3} \cdot \lim_{x \rightarrow 0} \frac{x^4}{\ln(1+x^4)}$$

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$$= \lim_{x \rightarrow 0} \frac{\cos x - x \sin x - \cos x}{3x^2} \cdot \lim_{x \rightarrow 0} \frac{4x^3}{4x^3 + 1+x^4}$$

$$= -\frac{1}{3} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} (1+x^4) = -\frac{1}{3}$$