*Do not forget to write your **full name** and your **Bilkent ID number**, and **sign** on the upper right corner of your paper.*

Final Exam Question 1.

Evaluate the following integrals:

a. $\displaystyle\int_{-1}^{1} (x^3 - 3x^2 + 2)(x^2 - 2x - 2)^{2021}\, dx$

b. $\displaystyle\int \cos 2x \tan^3 x\, dx$

Show all your work!
Explain your reasoning fully and in detail using correct mathematical notation and terminology, and in well-formed mathematical and English sentences!

a. $$\int_{-1}^{1} \underbrace{(x^3-3x^2+2)}_{(x-1)\cdot(x^2-2x-2)} \cdot (x^2-2x-2)^{2021}\, dx = \int_{-1}^{1} (x^2-2x-2)^{2022}\cdot(x-1)\, dx$$

$$= \int_{1}^{-3} u^{2022}\cdot \frac{1}{2}\, du = \frac{1}{2}\cdot \frac{u^{2023}}{2023}\Bigg]_{1}^{-3} = -\frac{3^{2023}+1}{4046}$$

$\boxed{\begin{array}{l} u = x^2-2x-2 \\ du = 2(x-1)\,dx \end{array}}$

b. $$\int \cos 2x \tan^3 x\, dx = \int (2\cos^2 x - 1)\cdot \frac{\sin^3 x}{\cos^3 x}\, dx = \int (2\cos^2 x - 1)\cdot\frac{(1-\cos^2 x)}{\cos^3 x}\cdot \sin x\, dx$$

$$= \int (2u^2-1)\cdot\frac{(1-u^2)}{u^3}\cdot(-du) = \int \left(2u - \frac{3}{u} + \frac{1}{u^3}\right) du = u^2 - 3\ln|u| - \frac{1}{2u^2} + C$$

$\boxed{\begin{array}{l} u = \cos x \\ du = -\sin x\, dx \end{array}}$

$$= \cos^2 x - 3\ln|\cos x| - \frac{1}{2}\sec^2 x + C$$