3. Suppose that f is a differentiable function and $g(x) = f(xf(x^2) + f(x))$.

Find $f(5)$ and $f'(5)$ if

❶ $f(2) = 3$, $f(3) = 5$, $f(4) = 1$, $f'(2) = -2$, $f'(3) = -4$, $f'(4) = -3$; and

❷ $y = 20x + 19$ is an equation for the tangent line to the graph of $y = g(x)$ at the point with $x = 2$.

② $\Rightarrow$ $g(2) = y|_{x=2} = (20x+19)|_{x=2} = 20 \cdot 2 + 19 = 59$

$\Vert$

① $\Rightarrow$ $f(2f(4)+f(2)) = f(2 \cdot 1 + 3) = f(5)$

$\rightarrow$ $f(5) = 59$

$g'(x) = f'(xf(x^2)+f(x)) \cdot (1 \cdot f(x^2) + x \cdot f'(x^2) \cdot 2x + f'(x))$

$\Downarrow$

① $\Rightarrow$ $g'(2) = f'(2f(4)+f(2)) \cdot (f(4) + 8f'(4) + f'(2))$

$= f'(2 \cdot 1 + 3) \cdot (1 + 8 \cdot (-3) + (-2))$

$= -25 f'(5)$

② $\Rightarrow$ $g'(2) = 20$

$\rightarrow$ $f'(5) = -\frac{4}{5}$