

1. Find  $y'''|_{x=1}$  if  $y$  is a differentiable function of  $x$  satisfying the identity  $y^3 + xy + x^2 = 1$ .

$$\begin{aligned} \boxed{x=1} \\ y^3 + y + 1 &= 1 \\ \Downarrow \\ y(y^2 + 1) &= 0 \\ \Downarrow \\ y &= 0 \end{aligned}$$

$$3y^2y' + y + xy' + 2x = 0$$

$$\boxed{x=1, y=0}$$

$$\begin{aligned} 0 + 0 + y' + 2 &= 0 \\ \Downarrow \\ y' &= -2 \end{aligned}$$

$$6y(y')^2 + 3y^2y'' + y' + y' + xy'' + 2 = 0$$

$$\boxed{x=1, y=0, y'=-2}$$

$$\begin{aligned} 0 + 0 + (-2) + (-2) + y'' + 2 &= 0 \\ \Downarrow \\ y'' &= 2 \end{aligned}$$

$$6(y')^3 + 12yy'y'' + 6yy'y'' + 3y^2y''' + 2y'' + y'' + xy''' = 0$$

$$\boxed{x=1, y=0, y'=-2, y''=2}$$

$$-48 + 0 + 0 + 0 + 4 + 2 + y''' = 0$$

$$\Downarrow \\ y''' = 42 \quad \text{when } x=1$$