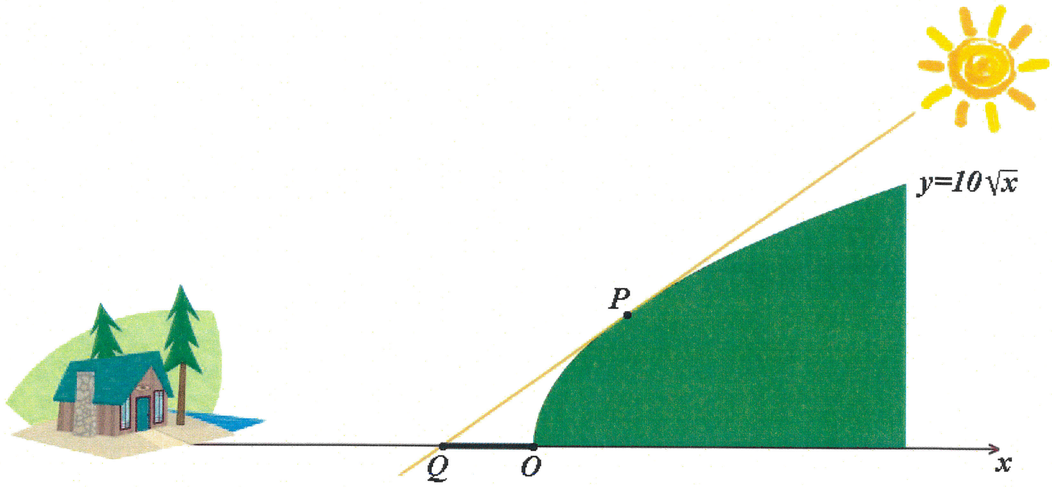
4. You have a cabin on the negative x-axis. A hill whose height is given by $y = 10\sqrt{x}$ for $x \geq 0$ lies to the west along the positive x-axis. (All coordinates are measured in meters.) As the sun starts to set, the hill casts a shadow as shown in the figure. Determine how fast the shadow is approaching your cabin at the moment when the sunrays are making a 30° angle with the horizontal and this angle is decreasing at a rate of 1/4 °/min. Express your answer in units of meters per minute.



Let a be the x-coordinate of the point P where the sunray is tangent to the hill, and let θ be the angle between the sunray and the positive x-axis.

$$y' = \frac{10}{2\sqrt{x}} = \frac{5}{\sqrt{x}} \Rightarrow \tan\theta = (\text{The slope of the ray}) = y'\big|_{x=a} = \frac{5}{\sqrt{a}} \Rightarrow a = \frac{25}{\tan^2\theta}$$

The equation of the tangent line is: $y - 10\sqrt{a} = \frac{5}{\sqrt{a}}\cdot(x-a)$

Hence, $y=0 \Rightarrow -10\sqrt{a} = \frac{5}{\sqrt{a}}\cdot(x-a) \Rightarrow x=-a$ is the x-coordinate of Q.

$$|QO| = a = \frac{25}{\tan^2\theta} \Rightarrow \frac{d}{dt}|QO| = -2\cdot\frac{25}{\tan^3\theta}\cdot\sec^2\theta\cdot\frac{d\theta}{dt}$$

When $\theta = 30° = \frac{\pi}{6}$ and $\frac{d\theta}{dt} = -\frac{1}{4}$ °/min $= -\frac{1}{4}\cdot\frac{\pi}{180}$ 1/min, this gives:

$$\frac{d}{dt}|QO| = -2\cdot\frac{25}{\tan^3\frac{\pi}{6}}\cdot\sec^2\frac{\pi}{6}\cdot\left(-\frac{1}{4}\cdot\frac{\pi}{180}\right) = +2\cdot\frac{25}{\left(\frac{1}{\sqrt{3}}\right)^3}\cdot\left(\frac{2}{\sqrt{3}}\right)^2\cdot\frac{1}{4}\cdot\frac{\pi}{180} = \frac{5\pi}{6\sqrt{3}} \text{ m/min}$$

The shadow is approaching the cabin with a speed of $\frac{5\pi}{6\sqrt{3}}$ m/min at that moment.