3. Find $\left.\dfrac{d^2y}{dx^2}\right|_{(x,y)=(\pi/2\,,\,\pi/6)}$ if y is a differentiable function of x satisfying the equation:

$$2\sin^2(x+y) = \sin x + \sin y$$

$$\Downarrow$$

$$1-\cos(2\cdot(x+y)) = \sin x + \sin y$$

$$\Downarrow d/dx$$

$$\sin(2\cdot(x+y))\cdot 2\cdot(1+y') = \cos x + \cos y\cdot y'$$

$$\Downarrow (x,y)=\left(\tfrac{\pi}{2},\tfrac{\pi}{6}\right)$$

$$\sin\left(\tfrac{4\pi}{3}\right)\cdot 2\cdot(1+y') = \cos\tfrac{\pi}{2} + \cos\tfrac{\pi}{6}\cdot y'$$

$$\Downarrow$$

$$-\tfrac{\sqrt{3}}{2}\cdot 2\cdot(1+y') = 0 + \tfrac{\sqrt{3}}{2}\cdot y'$$

$$\Downarrow$$

$$y' = -\tfrac{2}{3} \text{ at } (x,y)=\left(\tfrac{\pi}{2},\tfrac{\pi}{6}\right)$$

d/dx

$$\cos(2\cdot(x+y))\cdot 4\cdot(1+y')^2 + \sin(2\cdot(x+y))\cdot 2\cdot y'' = -\sin x - \sin y\cdot(y')^2 + \cos y\cdot y''$$

$$\Downarrow (x,y)=\left(\tfrac{\pi}{2},\tfrac{\pi}{6}\right),\ y'=-\tfrac{2}{3}$$

$$\cos\left(\tfrac{4\pi}{3}\right)\cdot 4\cdot\left(1+\left(-\tfrac{2}{3}\right)\right)^2 + \sin\left(\tfrac{4\pi}{3}\right)\cdot 2\cdot y'' = -\sin\tfrac{\pi}{2} - \sin\tfrac{\pi}{6}\cdot\left(-\tfrac{2}{3}\right)^2 + \cos\tfrac{\pi}{6}\cdot y''$$

$$\Downarrow$$

$$-\tfrac{1}{2}\cdot 4\cdot\tfrac{1}{9} + \left(-\tfrac{\sqrt{3}}{2}\right)\cdot 2\cdot y'' = -1 - \tfrac{1}{2}\cdot\tfrac{4}{9} + \tfrac{\sqrt{3}}{2}y''$$

$$\Downarrow$$

$$y'' = \frac{2}{3\sqrt{3}} \text{ at } (x,y)=\left(\tfrac{\pi}{2},\tfrac{\pi}{6}\right)$$