1. In each of the following, if the given statement is true for all functions f that are defined on $(-\infty, \infty)$, then mark the ☐ to the left of TRUE with a ✗; otherwise, mark the ☐ to the left of FALSE with a ✗ and give a counterexample.

a. If $f(2x) = f(x)$ for all x, then f is constant on $(-\infty, \infty)$.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases}$

b. If f is continuous on $(-\infty, \infty)$, then f has a derivative on $(-\infty, \infty)$.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) = |x|$

c. If f is continuous on $(-\infty, \infty)$, then f has an antiderivative on $(-\infty, \infty)$.

☒ TRUE

☐ FALSE, because it does not hold for $f(x) =$

d. If f is continuous on $(-\infty, \infty)$, then $\int f(x)\,dx = \frac{1}{2} f(x)^2 + C$ on $(-\infty, \infty)$.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) = 1$

e. If $\int_{-1}^{1} f(x)\,dx = 0$, then $\int_{-1}^{1} f(x)^3\,dx = 0$.

☐ TRUE

☒ FALSE, because it does not hold for $f(x) = \begin{cases} -1 & \text{if } x < \frac{1}{2} \\ 3 & \text{if } x \geq \frac{1}{2} \end{cases}$