4. At a certain moment the sides of a triangle are changing as follows:

❶ The length of the first side is 3 m and increasing at a rate of 5 m/s.

❷ The length of the second side is 8 m and increasing at a rate of 2 m/s.

❸ The length of the third side is 7 m and increasing at a rate of 1 m/s.

Determine how fast the angle between the first two sides is changing at this moment. Express your answer in units of °/s.

a = length of first side
b = length of second side
c = length of third side
θ = angle between the first two sides

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$a = 3m,\ b = 8m,\ c = 7m$

$$49 = 9 + 64 - 48\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$

$\frac{d}{dt}$

$$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} - 2\frac{da}{dt}b\cos\theta - 2a\frac{db}{dt}\cos\theta - 2ab\cdot(-\sin\theta)\cdot\frac{d\theta}{dt}$$

$a = 3m,\ b = 8m,\ c = 7m$
$\frac{da}{dt} = 5\,m/s,\ \frac{db}{dt} = 2\,m/s,\ \frac{dc}{dt} = 1\,m/s$

$$7\cdot 1 = 3\cdot 5 + 8\cdot 2 - 5\cdot 8\cdot\frac{1}{2} - 3\cdot 2\cdot\frac{1}{2} + 3\cdot 8\cdot\frac{\sqrt{3}}{2}\cdot\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = -\frac{1}{12\sqrt{3}}\ rad/s = -\frac{1}{12\sqrt{3}}\cdot\frac{180}{\pi}\ °/s = -\frac{5\sqrt{3}}{\pi}\ °/s$$

The angle between the first two sides is decreasing at a rate of $\frac{5\sqrt{3}}{\pi}$ °/s at this moment.