3. A function f satisfies the following conditions:

❶ f is differentiable on $(-\infty, \infty)$.

❷ $f(x) = 5x + 4$ for $x \le -1$.

❸ $f(x) = x$ for $x \ge 1$.

a. Show that there is a real number A such that $f(A) = 0$.

$f(1) = 1 > 0 > -1 = f(-1)$

Since f is diff'ble, f is continuous on $[-1, 1]$.

Therefore, there is A in $(-1, 1)$ with $f(A) = 0$ by IVT.

b. Show that there is a real number B such that $f'(B) = 2$.

f is diff'ble, hence continuous on $(-\infty, \infty)$. In particular, f is continuous on $[-2, 2]$ and diff'ble on $(-2, 2)$.

Therefore, there is B in $(-2, 2)$ such that

$$f'(B) = \frac{f(2) - f(-2)}{2 - (-2)} = \frac{2 - (-6)}{4} = 2 \text{ by MVT.}$$

c. Give an example of a function satisfying the conditions ❶, ❷ and ❸ by explicitly defining $f(x)$ for $-1 < x < 1$. *(There are many such functions. You do not have to explain how you found your example, but you must verify that it satisfies the conditions.)*

Let $f(x) = x^3 - x^2 + 1$ for $-1 < x < 1$. Then

$(x^3 - x^2 + 1)\big|_{x=1} = 1 = x\big|_{x=1}$ and $\frac{d}{dx}(x^3 - x^2 + 1)\big|_{x=1} = 1 = \frac{d}{dx}x\big|_{x=1}$, and

$(x^3 - x^2 + 1)\big|_{x=-1} = -1 = (5x+4)\big|_{x=-1}$ and $\frac{d}{dx}(x^3 - x^2 + 1)\big|_{x=-1} = 5 = \frac{d}{dx}(5x+4)\big|_{x=-1}$

Hence f is diff'ble on $(-\infty, \infty)$.