1. Find $\left.\dfrac{d^2y}{dx^2}\right|_{(x,y)=(2\,,1/2)}$ if y is a differentiable function of x satisfying the equation:

$$\sin(\pi x y) = \sin(\pi x) + \cos(\pi y)$$

$$\Big\Downarrow \ d/dx$$

$$\cos(\pi xy)\cdot\pi\cdot(y+xy') = \cos(\pi x)\cdot\pi - \sin(\pi y)\cdot\pi\cdot y'$$

$$\Big\Downarrow \leftarrow \boxed{(x,y)=(2,\tfrac{1}{2})}$$

$$\cos(\pi\cdot 2\cdot\tfrac{1}{2})\cdot\pi\cdot(\tfrac{1}{2}+2y') = \cos(\pi\cdot 2)\cdot\pi - \sin(\pi\cdot\tfrac{1}{2})\cdot\pi\cdot y'$$

$$\Big\Downarrow$$

$$-\pi\cdot(\tfrac{1}{2}+2y') = \pi - \pi\cdot y'$$

$$\Big\Downarrow$$

$$y' = -\frac{3}{2} \quad \text{at } (x,y)=(2,\tfrac{1}{2})$$

d/dx

$$-\sin(\pi xy)\cdot\pi^2\cdot(y+xy')^2 + \cos(\pi xy)\cdot\pi\cdot(y'+y'+xy'')$$

$$= -\sin(\pi x)\cdot\pi^2 - \cos(\pi y)\cdot\pi^2\cdot(y')^2 - \sin(\pi y)\cdot\pi\cdot y''$$

$$\Big\Downarrow \leftarrow \boxed{(x,y)=(2,\tfrac{1}{2}),\ y'=-\tfrac{3}{2}}$$

$$-\sin(\pi\cdot 2\cdot\tfrac{1}{2})\cdot\pi^2\cdot\left(\tfrac{1}{2}+2\cdot(-\tfrac{3}{2})\right)^2 + \cos(\pi\cdot 2\cdot\tfrac{1}{2})\cdot\pi\cdot(2\cdot(-\tfrac{3}{2})+2y'')$$

$$= -\sin(\pi\cdot 2)\cdot\pi^2 - \cos(\pi\cdot\tfrac{1}{2})\cdot\pi^2\cdot\left(-\tfrac{3}{2}\right)^2 - \sin(\pi\cdot\tfrac{1}{2})\cdot\pi\cdot y''$$

$$\Big\Downarrow$$

$$y'' = 3 \quad \text{at } (x,y)=(2,\tfrac{1}{2})$$