4a. Find $y(4)$ if $\dfrac{dy}{dx} = \sqrt{xy}$ and $y(0) = \dfrac{1}{9}$.

$$\Downarrow$$

$$\frac{dy}{\sqrt{y}} = \sqrt{x}\,dx \overset{\int}{\Longrightarrow} \underbrace{2\sqrt{y} = \frac{x^{3/2}}{3/2} + C' \xrightarrow{\boxed{x=0,\ y=\frac{1}{9}}} \frac{2}{3} = C'}$$

$$\Downarrow$$

$$y = \left(\frac{x^{3/2}+1}{3}\right)^2$$

$$\Downarrow$$

$$y(4) = \left(\frac{4^{3/2}+1}{3}\right)^2 = 9$$

4b. Suppose f is a continuous function such that

- $f(0) = 2017$, $f(2) = 2020$, and $f(4) = 2023$, and
- the average value of f on the interval $[0, 4]$ is 1.

Find $g'(2)$ where $g(x) = \int_0^x f(xt)\,dt$.

$$g(x) = \int_0^x f(xt)\,dt \underset{\boxed{\substack{u = xt \\ du = x\,dt}}}{=} \int_0^{x^2} f(u)\cdot\left(\frac{1}{x}\,du\right) = \frac{1}{x}\int_0^{x^2} f(u)\,du$$

$$g'(x) = \frac{d}{dx}\left(\frac{1}{x}\int_0^{x^2} f(u)\,du\right) = -\frac{1}{x^2}\int_0^{x^2} f(u)\,du + \frac{1}{x}\cdot\frac{d}{dx}\left(\int_0^{x^2} f(u)\,du\right)$$

$$= -\frac{1}{x^2}\int_0^{x^2} f(u)\,du + \frac{1}{x}\cdot f(x^2)\cdot\frac{d(x^2)}{dx} = -\frac{1}{x^2}\int_0^{x^2} f(u)\,du + 2f(x^2)$$

(Leibniz Rule)

$$g'(2) = -\underbrace{\frac{\int_0^4 f(u)\,du}{4}}_{\text{average of } f \text{ on } [0,4]} + 2f(4) = -1 + 2\cdot 2023 = 4045$$