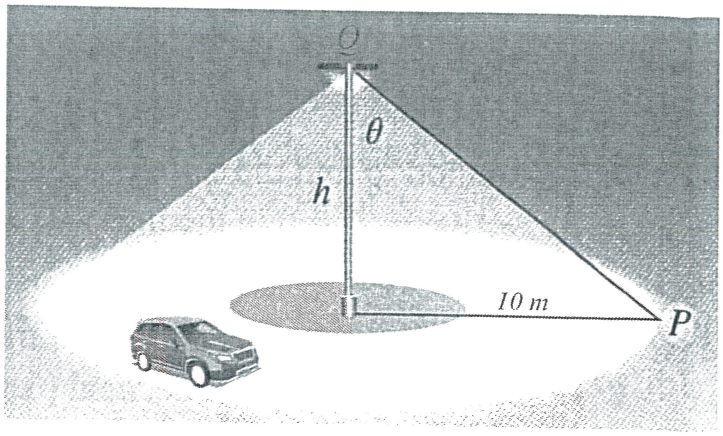
2. A lamp post will be erected in the center of a traffic circle with radius $10\,m$. The municipal regulations require the height h of the lamp post to be between $5\,m$ and $10\,m$. The intensity of illumination I at a point P at the edge of the traffic circle is given by

$$I = c\,\frac{\cos\theta}{(\text{the distance from } Q \text{ to } P)^2}$$

where c is a positive constant, and the point Q and the angle θ are as shown in the figure.



Determine how tall the lamp post should be to provide (a) the maximum and (b) the minimum illumination at P.

$|QP|^2 = h^2 + 10^2$ and $\cos\theta = \dfrac{h}{(h^2+10^2)^{1/2}}$.

Maximize/Minimize $I = c\cdot\dfrac{h}{(h^2+10^2)^{3/2}}$ for $5 \le h \le 10$

Critical points:

$$\frac{dI}{dh} = c\cdot\frac{1\cdot(h^2+10^2)^{3/2} - h\cdot\frac{3}{2}\cdot(h^2+10^2)^{1/2}\cdot 2h}{(h^2+10^2)^3} = c\cdot\frac{10^2-2h^2}{(h^2+10^2)^{5/2}}$$

$\dfrac{dI}{dh} = 0 \Rightarrow 10^2 - 2h^2 = 0 \Rightarrow h = 5\sqrt{2}$ or $h = -5\sqrt{2}$ ⊗ (not in the interval)

$\Downarrow$

$$I = \frac{c}{150\sqrt{3}}$$

Endpoints:

$h = 5 \Rightarrow I = \dfrac{c}{125\sqrt{5}}$

$h = 10 \Rightarrow I = \dfrac{c}{200\sqrt{2}}$

$128 > 125 > 108 \Rightarrow 8\sqrt{2} > 5\sqrt{5} > 6\sqrt{3} \Rightarrow \dfrac{1}{200\sqrt{2}} < \dfrac{1}{125\sqrt{5}} < \dfrac{1}{150\sqrt{3}}$

$\Rightarrow$ { A $5\sqrt{2}$ m tall lamp provides the maximum illumination.
A 10 m tall lamp provides the minimum illumination.