

1. Sketch the graph of $y = x^{1/3} + x^{-2/3}$ by computing y' and y'' , and determining their signs; finding the critical points, the inflections points, the intercepts, and the asymptotes; and clearly labeling them in the picture.

$$y' = \frac{1}{3} x^{-2/3} - \frac{2}{3} x^{-5/3} = \frac{1}{3} x^{-5/3} (x-2) = 0 \Rightarrow x=2$$

← undefined when $x=0$.

$$y'' = -\frac{2}{9} x^{-5/3} + \frac{10}{9} x^{-8/3} = -\frac{2}{9} x^{-8/3} (x-5) = 0 \Rightarrow x=5$$

x	0	2	5
y'	+	-	+
y''	+	+	-

$$\lim_{x \rightarrow 0} y = \infty$$

$$x=2 \Rightarrow y = 3 \cdot 2^{-2/3}$$

$$x=5 \Rightarrow y = 6 \cdot 5^{-2/3}$$

$$y=0 \Rightarrow x^{1/3} + x^{-2/3} = 0$$

$$\Rightarrow x^{-2/3} (x+1) = 0 \Rightarrow x = -1$$

