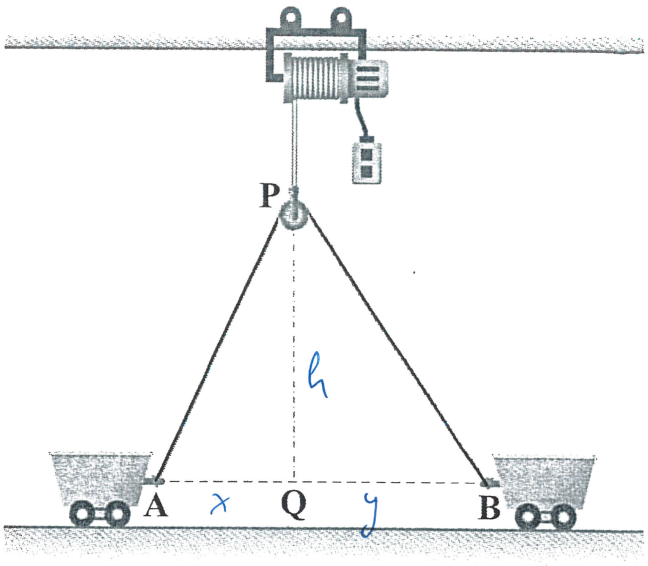
4. A 13 m long rope is tied to two carts at the points A and B, and passes over a pulley at the point P as shown in the figure. A winch moves the pulley along a fixed vertical line. The point Q lies directly below the point P and on the line connecting the points A and B.



At a certain moment when the winch is raising the pulley at a rate of 1 m/s; P is 4 m away from Q, A is 3 m away from Q, and the distance between the points A and Q is decreasing at a rate of 2 m/s.

Determine how fast the distance between the points B and Q is changing at this moment.

[Do not assume anything about how the quantities in the question depend on time beyond what is already given in the question.]

At our moment: $x = 3\,m$, $\frac{dx}{dt} = -2\,m/s$, $h = 4\,m$, $\frac{dh}{dt} = 1\,m/s$

$$\sqrt{x^2+h^2} + \sqrt{y^2+h^2} = 13\,m \xrightarrow{\text{at our moment}} \sqrt{3^2+4^2} + \sqrt{y^2+4^2} = 13$$

$$\Downarrow$$

$$y = 4\sqrt{3}\,m$$

$$\Downarrow d/dt$$

$$\frac{2x\frac{dx}{dt} + 2h\frac{dh}{dt}}{2\sqrt{x^2+h^2}} + \frac{2y\frac{dy}{dt} + 2h\frac{dh}{dt}}{2\sqrt{y^2+h^2}} = 0$$

$$\Downarrow \leftarrow \text{at our moment}$$

$$\frac{3\cdot(-2) + 4\cdot 1}{5} + \frac{4\sqrt{3}\cdot\frac{dy}{dt} + 4\cdot 1}{8} = 0$$

$$\Downarrow$$

$$\frac{dy}{dt} = -\frac{1}{5\sqrt{3}}\,m/s$$

The distance between B and Q is decreasing at a rate of $\frac{1}{5\sqrt{3}}$ m/s at this moment.