[25 points]

3. Find $\left.\dfrac{d^2y}{dx^2}\right|_{(x,y)=(0,1)}$ if y is a differentiable function of x satisfying the equation:

$$xy^2 + \sin(\pi y) = x^3$$

$$\Big\Downarrow \; d/dx$$

$$y^2 + 2xy\frac{dy}{dx} + \pi\cos(\pi y)\frac{dy}{dx} = 3x^2$$

$$\swarrow \; (x,y)=(0,1)$$

$$1 + 0\cdot\frac{dy}{dx} + \pi\cdot(-1)\frac{dy}{dx} = 0$$

$$\Big\Downarrow$$

$$\frac{dy}{dx} = \frac{1}{\pi} \quad \text{at } (x,y)=(0,1)$$

$$\Big\Downarrow \; d/dx$$

$$2y\frac{dy}{dx} + 2y\frac{dy}{dx} + 2x\left(\frac{dy}{dx}\right)^2 + 2xy\frac{d^2y}{dx^2} - \pi^2\sin(\pi y)\left(\frac{dy}{dx}\right)^2 + \pi\cos(\pi y)\frac{d^2y}{dx^2} = 6x$$

$$\Big\Downarrow \; (x,y)=(0,1),\ \frac{dy}{dx}=\frac{1}{\pi}$$

$$2\cdot\frac{1}{\pi} + 2\cdot\frac{1}{\pi} + 0\cdot\left(\frac{1}{\pi}\right)^2 + 0\cdot\frac{d^2y}{dx^2} - 0\left(\frac{dy}{dx}\right)^2 + \pi\cdot(-1)\frac{d^2y}{dx^2} = 0$$

$$\Big\Downarrow$$

$$\frac{d^2y}{dx^2} = \frac{4}{\pi^2} \quad \text{at } (x,y)=(0,1)$$