

1. Evaluate the following limits.

[Do not use L'Hôpital's Rule!]

$$\text{a. } \lim_{x \rightarrow 1} \frac{2x^3 - 3x^2 + 1}{x^3 - 3x + 2} = \lim_{x \rightarrow 1} \frac{(x-1) \cdot (2x^2 - x - 1)}{(x-1) \cdot (x^2 + x - 2)}$$

$$= \lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{x^2 + x - 2} = \lim_{x \rightarrow 1} \frac{(x-1) \cdot (2x+1)}{(x-1) \cdot (x+2)} = \lim_{x \rightarrow 1} \frac{2x+1}{x+2} = \frac{2 \cdot 1 + 1}{1 + 2} = 1$$

$$\text{b. } \lim_{x \rightarrow 0^-} \frac{\sqrt{\cos(x) - \cos(2x)}}{x}$$

$$\lim_{x \rightarrow 0} \frac{\cos x - \cos 2x}{x^2} = \lim_{x \rightarrow 0} \frac{\cos x - (2\cos^2 x - 1)}{x^2} = \lim_{x \rightarrow 0} \frac{(1 - \cos x) \cdot (1 + 2\cos x)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2\left(\frac{x}{2}\right)}{x^2} \cdot \lim_{x \rightarrow 0} (1 + 2\cos x) = \frac{1}{2} \cdot \left(\lim_{x \rightarrow 0} \frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}} \right)^2 \cdot (1 + 2 \cdot 1) = \frac{1}{2} \cdot 1^2 \cdot 3 = \frac{3}{2}$$

$$\lim_{x \rightarrow 0^-} \frac{\sqrt{\cos x - \cos 2x}}{x} = \lim_{x \rightarrow 0^-} \left(-\sqrt{\frac{\cos x - \cos 2x}{x^2}} \right) = - \left(\lim_{x \rightarrow 0^-} \frac{\cos x - \cos 2x}{x^2} \right)^{1/2} = -\sqrt{\frac{3}{2}}$$