

1. Evaluate the following limit  $\lim_{x \rightarrow 0} \left( \frac{1}{2x^2} + \frac{1}{x^3} + \frac{1}{x^4} - \frac{e^{\sin x}}{x^4} \right)$ .

$$\lim_{x \rightarrow 0} \left( \frac{1}{2x^2} + \frac{1}{x^3} + \frac{1}{x^4} - \frac{e^{\sin x}}{x^4} \right) = \lim_{x \rightarrow 0} \frac{1+x+\frac{x^2}{2} - e^{\sin x}}{x^4}$$

$$\stackrel{\uparrow}{\text{L'H}} = \lim_{x \rightarrow 0} \frac{1+x - e^{\sin x} \cdot \cos x}{4x^3} = \lim_{x \rightarrow 0} \frac{1 - e^{\sin x} \cdot \cos^2 x + e^{\sin x} \cdot \sin x}{12x^2}$$

$$\stackrel{\uparrow}{\text{L'H}} = \lim_{x \rightarrow 0} \frac{-e^{\sin x} \cdot \cos^3 x + e^{\sin x} \cdot 2\cos x \sin x + e^{\sin x} \cdot \cos x \sin x + e^{\sin x} \cdot \cos x}{24x}$$

$$= \lim_{x \rightarrow 0} \left( \frac{1}{8} \cdot \frac{\sin x}{x} \cdot \cos x \cdot e^{\sin x} + \frac{1}{24} \cdot \frac{1 - \cos^2 x}{x} \cdot \cos x \cdot e^{\sin x} \right)$$

$$= \frac{1}{8} \cdot 1 \cdot 1 \cdot 1 + \lim_{x \rightarrow 0} \left( \frac{1}{24} \cdot \frac{\sin x}{x} \cdot \sin x \cdot \cos x \cdot e^{\sin x} \right)$$

$$= \frac{1}{8} + \frac{1}{24} \cdot 1 \cdot 0 \cdot 1 \cdot 1 = \frac{1}{8}$$