ELEMENTARY NUMBER THEORY

HOMEWORK 1

(1) Find an example of nonunique factorization in the monoid

$$M = \{1, 6, 11, 16, \ldots\}$$

of numbers of the form $5n + 1$.

(2) Prove that $8 \mid (n^2 - 1)$ for all odd $n \in \mathbb{N}$
   (a) using induction on n;
   (b) by computing $n^2 \bmod 8$ for all four "odd" residue classes modulo 8;
   (c) by looking at the factorizations $8 = 2 \cdot 4$ and $n^2 - 1 = (n-1)(n+1)$.

(3) Factor integers of the form $4n^2 + 1$ for small values of n. List the prime factors that occur and find a rule that describes them.

(4) Show that there are infinitely many primes of the form $p \equiv 2 \bmod 3$. (Hint: $N = 3p_1 \cdots p_n + 2$.)

(5) Take an ISBN from one of your books and verify that it is valid.