

ELEMENTARY NUMBER THEORY

SOME PROBLEMS

- (1) Prove that $\phi(p^2) = p(p - 1)$.
- (2) Show that $e^2 - 2f^2 \not\equiv 3, 5 \pmod{8}$.
- (3) Find a primitive root modulo 13.
- (4) Define unit, prime, irreducible, order of an element.
- (5) Prove that $\gcd(a, b) = \gcd(a, a + b)$.
- (6) Show that there are infinitely many primes $\equiv 2 \pmod{3}$.
- (7) Prove Fermat's Little Theorem
- (8) Compute $\gcd(3 + 4i, 6 + 6i)$.
- (9) Is $X^3 + x + 1$ a quadratic residue modulo $X^2 + 1$ in \mathbb{F}_7 ?