ELEMENTARY NUMBER THEORY

HOMEWORK 2

(1) Show that there are infinitely many primes of the form $p \equiv 3 \bmod 4$ by modifying Euclid's proof.

Why does this trick not work for primes $p \equiv 1 \bmod 4$?

(2) Prove that
  (a) $\gcd(ma, mb) = m \cdot \gcd(a, b)$;
  (b) $\gcd(a, b) = \gcd(a, a + b)$.

(3) Show that if $n = x^2 + 2y^2$ is odd, then $n \equiv 1, 3 \bmod 8$.

(4) Compute the last digit of 7^{100}.

(5) Observe that $217 \equiv 2 + 1 + 7 \equiv 1 \bmod 9$. Find a generalization and prove it.

The homework will be collected in class on Monday, February 21.