ELEMENTARY NUMBER THEORY

HOMEWORK 1

(1) Prove the cancellation law in $\mathbb{N}$: if $x, y, z \in \mathbb{N}$ satisfy $x + z = y + z$, then $x = y$.

(2) Consider the set $N = \{0, 1\}$ with successor function $s : N \longrightarrow N$ mapping $0 \longmapsto 1$ and $1 \longmapsto 0$. Show that this system satisfies all Peano axioms except one – which one?

(3) Show that $[r, s] * [t, u] = [rt, su]$ is not well defined on $\mathbb{Z}$.

(4) Prove that addition on $\mathbb{Z}$ is commutative.

(5) Consider the monoid $M = 2\mathbb{Z} \cup \{1\} = \{1, 2, 4, 6, 8, \ldots\}$. Show that M does not contain any prime, and find all irreducible elements in M.

The homework will be collected in class on Monday, February 14.