DISCRETE MATHEMATICS

PROBLEMS

More problems from last year's exams.

(1) Let $ABCD$ be a square with $|AB| = 1$. Show that if we select 101 points in the interior of this square, there are at least two whose distance is less than $\frac{1}{5\sqrt{2}}$.

(2) Determine the number of integer solutions to

$$x_1 + x_2 + x_3 + x_4 = 23,$$

where $2 \leq x_i \leq 7$ for all $1 \leq i \leq 4$.

(3) Find the coefficient of x^{33} in $(x^3 + x^5 + x^7 + x^9 + x^{11})^7$.

(4) Use generating functions to solve the recurrence sequence $a_0 = 0$, $a_1 = 2$, $a_n = 2a_{n-1} + 2a_{n-2}$.

(5) Find the generating function for the sequence $\{a_n\}$, where $a_0 = 0$ and $a_n = 1^2 + 2^2 + \ldots + n^2$ for $n \geq 1$.

(6) Find the generating function for the sequence $a_n = 3n + 2^n$ for $n \geq 0$.

(7) Find the coefficient of x^4 in $\frac{1}{(1-2x)^7}$.

(8) Find the generating function for the recurring sequence defined by $a_0 = 1$, $a_n - 2a_{n-1} = n$ for $n \geq 1$. Use this to give a formula for a_n.

(9) Solve the recurrence relation $a_0 = 4$, $a_1 = 7$, $a_n = 5a_{n-1} - 6a_{n-2}$.

(10) Solve the recurrence relation $a_0 = 1$, $a_n - 2a_{n-1} = 3^n$ for $n \geq 1$.