DISCRETE MATHEMATICS

PROBLEMS

(1) (Midterm 1, 2006) Find the number of arrangements of all the letters in MASSASAUGA in which all the S's are together.

(2) (Midterm 1, 2006) In how many ways can 10 (identical) candy bars be distributed among five children so that the youngest gets only one or two of them?

(3) (Midterm 1, 2006) Define a sequence $a_0, a_1, \ldots$ recursively by $a_0 = 1$, $a_1 = 1$, $a_2 = 1$ and $a_n = a_{n-1} + a_{n-3}$ for $n \geq 3$. Show that $a_{n+2} \geq \sqrt{2}^n$ for all $n \geq 0$.

(4) (Midterm 1, 2006) Let A and B be sets with $|A| = 10$ and $|B| = 18$. How many relations from A to B are there?

(5) (Midterm 1, 2006) Let A and B be sets with $|A| = 7$ and $|B| = 5$. How many functions $f : A \longrightarrow B$ are there with the property $|f(A)| = 3$?

(6) (Midterm 2, 2006) How many integral solutions $x_i \geq 0$ does $x_1 + x_2 + x_3 + x_4 = 12$ have?

(7) (Final 2006) How many integers $n > 5000000$ can we write using the digits $3, 4, 4, 5, 5, 6, 7$?

(8) (Final 2006) Use induction to prove that $2^{2n+1} + 1$ is divisible by 3 for all integers $n \geq 0$. [Can you give a direct proof, too?]