

## DISCRETE MATHEMATICS

### HOMEWORK 4

- (1) Check which of the following relations are equivalence relations:
  - (a) On the set  $L$  of all lines in the plane  $\mathbb{R}^2$ , call two lines  $\ell_1$  and  $\ell_2$  related ( $\ell_1 \mathcal{R} \ell_2$ ) if  $\ell_1$  is perpendicular to  $\ell_2$ .
  - (b) Define a relation  $\mathcal{R}$  on  $\mathbb{Z}$  by saying  $x \mathcal{R} y$  for integers  $x, y$  if  $x + y$  is even.
  - (c) Define a relation  $\mathcal{R}$  on  $\mathbb{Z}$  by saying  $x \mathcal{R} y$  for integers  $x, y$  if  $x + y$  is odd.
  - (d) Let  $T$  be the set of triangles in  $\mathbb{R}^2$ , and call two triangles related if they have an angle of the same measure (that is, the same size).
- (2) Draw the digraph with vertices  $\{a, b, c, d, e, f\}$  and edges  $\{(a, b), (a, d), (b, c), (b, e), (d, b), (d, e), (e, c), (e, f), (f, d)\}$ . Also determine the adjacency matrix of this digraph.
- (3) Let  $A$  be a set with 5 elements. How many relations from  $A$  to  $A$  are there? How many of them are symmetric?
- (4) For  $A = \mathbb{R}^2$ , define a relation  $\mathcal{R}$  on  $A$  by  $(x_1, y_1) \mathcal{R} (x_2, y_2)$  if  $x_1 = x_2$ . Check that  $\mathcal{R}$  is an equivalence relation, and describe the equivalence classes geometrically.