DISCRETE MATHEMATICS

HOMEWORK 4

(1) Take 101 points in the unit square. Show that three of them are vertices of a triangle with area at most 0.01.

(2) The set of all points (x, y) in the plane with integers x, y is called a lattice, and the points (x, y) are called lattice points. Show that among five lattice points, there are always two points such that the segment connecting them contains another lattice point.

(3) For each of the following functions $f : \mathbb{Z} \longrightarrow \mathbb{Z}$, determine whether the function is injective or surjective; determine its range (the image of f), and find out which of these functions have an inverse.
   (a) $f(x) = x + 7$
   (b) $f(x) = -x + 5$
   (c) $f(x) = x^2 + x$
   (d) $f(x) = 2x - 3$
   (e) $f(x) = x^2$
   (f) $f(x) = x^3$

(4) Find sets A, B, subsets $A_1, A_2 \subseteq A$, and a function $f : A \longrightarrow B$ with the property that $f(A_1 \cap A_2) \neq f(A_1) \cap f(A_2)$.