DISCRETE MATHEMATICS

(1) Find the coefficient of xyz^2 in $(2x - y - z)^4$.

(2) How many 10-digit numbers are there which use each digit exactly once, where no odd digit ever follows an even digit, and where the 4 and the 5 are next to each other?

(3) Assume you are given 15 points in the plane, no three of which are collinear (lie on the same line). How many lines do these points determine?

(4) Use induction to show that
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$
Can you also give a direct proof?

(5) Use induction to show that
$$1 + 2 + 4 + \ldots + 2^n = 2^{n+1} - 1.$$
Can you also give a direct proof?