DISCRETE MATHEMATICS

HOMEWORK 1

(1) Serial numbers on 5-dollar bills consist of two letters, eight digits, and another letter. How many different such numbers are there?

Just multiply the number of possible choices at each stage; the correct answer is $26^3 \cdot 10^8$.

(2) In how many ways can you put 6 horses into 10 stalls (at most one horse per stall) ?

For each of the 10 stalls, write down H if it contains a horse, and N if not. Then the number of ways is equal to the number of 10-letter words with 6 H's and 4 N's. This in turn is equal to $\frac{10!}{6!4!} = \binom{10}{4} = \binom{10}{6} = 210$.

(3) How many 11-letter words can you form out of the eleven letters of the word CHUMBAWUMBA ?

There are 11 letters, with 2 A's, 2 B's, 2 M's and 2 U's, so the number is $\frac{11!}{2!2!2!2!}$.

(4) Assume you throw a coin nine times. In how many ways can you get three heads and six tails?

Here each possibility is described by a 9-letter word made up from 3 H's and 6 T's; the answer is $\binom{9}{3} = 84$.

(5) Determine the number of integral solutions of the equation $x_1+x_2+x_3 = 40$, where $x_1 \geq -2$ and $x_2, x_3 \geq 0$.

The equation is equivalent to $x_1 + 2 + x_2 + x_3 = 42$; we are distributing 42 units to each of the variables $y_1 = x_1 + 2$, x_2, and x_3, so we are counting 44-letter words with 42 U's and 2 bars. Thus the answer is $\binom{44}{2} = 22 \cdot 43$.