ALGEBRAIC NUMBER THEORY

HOMEWORK 3

(1) Find all prime ideals of norm ≤ 7 in $\mathbb{Q}(\sqrt{19}\,)$.

(2) Use the elements $4+\sqrt{19}$ and $5+\sqrt{19}$ to find a unit in $\mathbb{Q}(\sqrt{19}\,)$. Show that it is fundamental.

(3) Show that the equation $x^2 - 2py^2 = -1$ is solvable for primes $p \equiv 5 \bmod 8$.

(4) Compute the fundamental unit of $\mathbb{Q}(\sqrt{199}\,)$ using elements of small norm.
Hint: The command

```
for(a=1,30,print(a,"   ",factor(a^2-199)))
```

will give you elements whose norms are divisible only by 2, 3, 5; use these.

(5) Show that $\mathbb{Q}(\sqrt{-43}\,)$ has class number 1.