

## ALGEBRAIC NUMBER THEORY

### HOMEWORK 1

- (1) Using the Euclidean algorithm in  $\mathbb{Z}[i]$ , find the gcd of the Gaussian integers  $10+11i$  and  $11+16i$ . Can you factor these elements into irreducible factors?
- (2) Show that  $\mathbb{Z}[\sqrt{2}]$  is Euclidean with respect to absolute value of the norm.
- (3) Show that  $10 = 2 \cdot 5 = -\sqrt{-10} \cdot \sqrt{-10}$  is an example of nonunique factorization in  $\mathbb{Z}[\sqrt{-2}]$ .
- (4) Show that  $6 = 2 \cdot 3 = (2 + \sqrt{-2})(2 - \sqrt{-2})$  is not an example of nonunique factorization in  $\mathbb{Z}[\sqrt{-2}]$ .
- (5) Let  $f$  be a Euclidean function on  $R$ . Show that if  $f(u) = 1$ , then  $u$  is a unit. (Hint: look at  $1 = uq + r$ .)