Alex Degtyarev

RECENT PAPERS WITH ABSTRACTS

1. A. Degtyarev, *On deformations of singular plane sextics*, J. Algeb. Geom. **17** (2008), 101–135.

Abstract: We study complex plane projective sextics with simple singularities up to equisingular deformations. It is shown that two such curves are deformation equivalent if and only if the corresponding pairs are diffeomorphic. A way to enumerate all deformation classes is outlined, and a few examples are considered, including classical Zariski pairs; in particular, promising candidates for homeomorphic but not diffeomorphic pairs are found.

2. A. Degtyarev, *Oka's conjecture on irreducible plane sextics*, J. London Math. Soc. **78** (2008), no. 2, 329–351, `doi:10.1112/jlms/jdn029`.
3. A. Degtyarev, *Oka's conjecture on irreducible plane sextics.* II, J. Knot Theory Ramifications **18** (2009), no. 8, 1065-1080, `doi:10.1142/S0218216509007348`.

Abstract: We partially prove and partially disprove Oka's conjecture on the Alexander polynomial/fundamental group of an irreducible plane sextic. As a by-product, all irreducible sextics admitting dihedral coverings are found.

4. A. Degtyarev, *Plane sextics via dessins d'enfants*, Geometry & Topology **14** (2010), no. 1, 393–433, `doi:10.2140/gt.2010.14.393`.
5. A. Degtyarev, *Plane sextics with a type* $\mathbf{E}_8$ *singular point*, Tohoku Math. J. **62** (2010), no. 3, 329–355, `doi:10.2748/tmj/1287148615`.
6. A. Degtyarev, *Plane sextics with a type* $\mathbf{E}_6$ *singular point*, Michigan Math. J. **60** (2011), 243–269, `doi:10.1307/mmj/1310667976`.
7. A. Degtyarev, *The fundamental group of a generalized trigonal curve*, Osaka J. Math. **48** (2011), no. 3, 749–782.

Abstract: We compute the fundamental groups of most irreducible sextics with a triple singular point, using the relation between such sextics and trigonal curves and the techniques of dessins d'enfants.

8. A. Degtyarev, *Stable symmetries of plane sextics*, Geometriæ Dedicata **137** (2008), no. 1, 199–218, `doi:10.1007/s10711-008-9293-6`.
9. A. Degtyarev, *Fundamental groups of symmetric sextics*, J. Math. Kyoto Univ. **48** (2008), no. 4, 765–792.
10. A. Degtyarev, *Irreducible plane sextics with large fundamental groups*, J. Math. Soc. Japan **61** (2009), no. 4, 1131–1169.
11. A. Degtyarev, *On irreducible sextics with non-abelian fundamental group*, Proceedings of Niigata–Toyama Conferences 2007, Adv. Stud. Pure Math., vol. 56, 2009, pp. 65–92.
12. A. Degtyarev, M. Oka, *A plane sextic with finite fundamental group*, Proceedings of Niigata–Toyama Conferences 2007, Adv. Stud. Pure Math., vol. 56, 2009, pp. 93–108.
13. A. Degtyarev, *Fundamental groups of symmetric sextics.* II, Proc. London Math. Soc., 99:2 (2009), 353–385, `doi:10.1112/plms/pdp003`.
14. A. Degtyarev, *Classical Zariski pairs*, J. Singularities, 2 (2010), 51–55.

Abstract: We classify and compute the fundamental groups of all irreducible plane sextics admitting a stable (with respect to equisingular deformations) symmetry. Together with [4]–[7], these

These abstracts, roughly grouped by projects, represent my current research interests
Most papers are available at `http://fen.bilkent.edu.tr/~degt/`

results cover most irreducible sextics of torus type, as well as other sextics admitting dihedral coverings, see [2], [3].

15. A. Degtyarev, *Zariski k-plets via dessins d'enfants*, Comment. Math. Helv. **84** (2009), no. 3, 639–671, `doi:10.4171/CMH/176`.
16. A. Degtyarev, *Transcendental lattice of an extremal elliptic surface*, J. Algeb. Geom. **21** (2012), 413–444, `doi:10.1090/S1056-3911-2011-00563-8`.
17. A. Degtyarev, *Hurwitz equivalence of braid monodromies and extremal elliptic surfaces*, Proc. London Math. Soc. (3) **103** (2011), 1083–1120, `doi:10.1112/plms/pdr013`.
18. A. Degtyarev, N. Salepci, *Products of pairs of Dehn twists and maximal real Lefschetz fibrations*, Nagoya Math. J. **210** (2013), 83–132, `doi:10.1215/00277630-2077026`.

Abstract: We exploit the close relation between trigonal curves/elliptic surfaces, skeletons (certain bipartite ribbon graphs), and subgroups of the modular group $\Gamma := PSL(2,\mathbb{Z})$. The principal results are: (1) the equivalence of topological and analytic classifications of extremal elliptic surfaces and maximal trigonal curves; (2) exponentially large examples of nonequivalent surfaces, curves, and, most striking, simple monodromy factorizations of the same element at infinity in the simplest nonabelian braid group $\mathbb{B}_3$; (3) computation of various invariants of elliptic surfaces and trigonal curves in combinatorial terms; (4) the classification of $\mathbb{B}_3$-valued monodromy factorizations of length two. We also discuss applications to real trigonal curves and real Lefschetz fibrations.

19. A. Degtyarev, *Dihedral coverings of trigonal curves*, Indiana Univ. Math. J. **61** (2012), no. 3, 901–938, `doi:10.1512/iumj.2012.61.4607`.
20. A. Degtyarev, *The Alexander module of a trigonal curve*, Rev. Mat. Iberoam. **30** (2014), no. 1, 25–64, `doi:10.4171/RMI/768`.
21. A. Degtyarev, *The Alexander module of a trigonal curve. II*, Singularities in geometry and topology 2011, Adv. Stud. Pure Math., vol. 66, Math. Soc. Japan, Tokyo, 2015, pp. 47–69.

Abstract: As a continuation of [15]–[18], we begin a systematic study of the fundamental group π_1 of a trigonal curve and, as a consequence, of a plane curve with a singular point of multiplicity $\deg - 3$. As a first step, we compute the metabelian invariants (dihedral quotients and the Alexander module). With few sporadic exceptions, these invariants are controlled by congruence subgroups of the modular group. We discover a close similarity of trigonal curves and plane sextics: there are strong restrictions to π_1 and the existence of certain quotients of π_1 may imply various geometric properties of the curve, such as torus type (Oka's conjecture in the case of sextics) or the existence of splitting sections.

22. A. Degtyarev, *Topology of Algebraic Curves: An Approach via Dessins d'Enfants*, De Gruyter Studies in Mathematics, vol. 44, Walter de Gruyter & Co., Berlin, 2012, pp. xvi+393, ISBN: 978-3-11-025591-1.

Abstract: This monograph unifies and completes the results of [4]–[7], [15]–[18], and [19]–[21]. In particular, we complete the deformation classification and compute the fundamental groups of all irreducible plane sextics with an at least triple singular point. Among other results are *universal* (*i.e.,* not depending on the singularities of the curve or its degree) bounds on the metabelian invariants of a trigonal curve, the concept of *universal trigonal curves* and, as a result, various relations between the monodromy/fundamental group of a curve and its geometric properties (torus type, splitting sections, *etc.*), numerous examples of exponentially large Zariski k-plets (in various settings), and a few steps towards the further understanding of the Hurwitz equivalence of monodromy factorizations taking values in the modular group $\Gamma := PSL(2,\mathbb{Z})$ or braid group $\mathbb{B}_3$.

23. A. Degtyarev, *On plane sextics with double singular points*, Pacific J. Math. **265** (2013), no. 2, 327–348, `doi:10.2140/pjm.2013.265.327`.

24. A. Degtyarev, *On the Artal–Carmona–Cogolludo construction*, J. Knot Theory Ramifications **23** (2014), no. 5, 1450028 (35 pages), doi:10.1142/S021821651450028X.

Abstract: These papers start a systematic study of tetragonal curves in ruled surfaces, the principal goal being the computation of the fundamental groups of the maximizing irreducible plane sextics with **A**-type singularities only. (There are a few dozens of such curves whose groups are still unknown.) We obtain explicit defining equations of most sextics and, as a first step, compute the groups of most real curves. In particular, the groups of all but two irreducible curves of torus type are found. (The two remaining curves are not real.) As an outcome, we obtain a few interesting observations concerning the minimal field of definition of sextics and construct several homotopy equivalent arithmetic Zariski pairs.

25. A. Degtyarev, T. Ekedahl, I. Itenberg, B. Shapiro, M. Shapiro, *On total reality of meromorphic functions*, Ann. Inst. Fourier **57** (2007), no. 5, 2015–2030.
26. A. Degtyarev, *Towards the generalized Shapiro and Shapiro conjecture*, Perspectives in analysis, geometry, and topology, Progr. Math., vol. 296, Birkhäuser/Springer, New York, 2012, pp. 67–79, doi:10.1007/978-0-8176-8277-4_4.

Abstract: We prove the generalized Shapiro and Shapiro conjecture in a few special cases. The conjecture states that a meromorphic function on a compact real curve with all critical points real is real with respect to an appropriate real structure in the target.

27. A. Degtyarev, I. Itenberg, V. Kharlamov, *On deformation types of real elliptic surfaces*, Amer. J. Math. **130** (2008), no. 6, 1561–1627, doi:10.1353/ajm.0.0029.
28. A. Degtyarev, I. Itenberg, V. Zvonilov, *Real trigonal curves and real elliptic surfaces of type* I, J. Reine Angew. Math. **686** (2014), 221–246, doi:10.1515/crelle-2012-0020.

Abstract: We study real elliptic surfaces and trigonal curves (over a base of an arbitrary genus) and their equivariant deformations. We calculate the real Tate–Shafarevich group and reduce the deformation classification to the combinatorics of a real version of Grothendieck's *dessins d'enfants*. As a consequence, we obtain an explicit description of the deformation classes of M- and $(M-1)$- (*i.e.*, maximal and submaximal in the sense of the Smith inequality) curves and surfaces, as well as a partial classification of maximally inflected separating real curves and real elliptic surfaces of type I.

29. A. Degtyarev, I. Itenberg, V. Kharlamov, *On the number of components of a complete intersection of real quadrics*, Perspectives in analysis, geometry, and topology, Progr. Math., vol. 296, Birkhäuser/Springer, New York, 2012, pp. 81–107, doi:10.1007/978-0-8176-8277-4_5.

Abstract: Our main results concern complete intersections of three real quadrics. We prove that the maximal number $B_2^0(N)$ of connected components that a regular complete intersection of three real quadrics in $\mathbb{P}^N$ can have differs at most by one from the maximal number of ovals of the submaximal depth $[(N-1)/2]$ of a real plane projective curve of degree $d = N+1$. As a consequence, we obtain a lower bound $\frac{1}{4}N^2 + O(N)$ and an upper bound $\frac{3}{8}N^2 + O(N)$ for $B_2^0(N)$.

30. A. Degtyarev, I. Itenberg, *On real determinantal quartics*, Proceedings of the Gökova Geometry–Topology Conference 2010, Int. Press, Somerville, MA, 2011, pp. 110–128.

Abstract: We describe all possible arrangements of the ten nodes of a generic real determinantal quartic surface in $\mathbb{P}^3$ with nonempty spectrahedral region.

31. A. Akyol, A. Degtyarev, *Geography of irreducible plane sextics*, Proc. London Math. Soc. (3) **111** (2015), 1307–1337, doi:10.1112/plms/pdv053.

Abstract: We complete the equisingular deformation classification of irreducible singular plane sextic curves. We also describe the monodromy groups of the strata and discuss whether real strata are represented by real curves. As a by-product, we compute the fundamental groups of the complement of all but a few maximizing sextics.

32. A. Degtyarev, *Lines generate the Picard groups of certain Fermat surfaces*, J. Number Theory **147** (2015), 454–477, doi:10.1016/j.jnt.2014.07.020.
33. A. Degtyarev, *On the Néron–Severi lattice of a Delsarte surface*, Kyoto J. Math. **56** (2016), no. 3, 611–632, doi:10.1215/21562261-3600202.
34. A. Degtyarev, I. Shimada, *On the topology of projective subspaces in complex Fermat varieties*, J. Math. Soc. Japan. **68** (2016), no. 3, 975–996, doi:10.2969/jmsj/06830975.
35. A. Degtyarev, *Projective spaces in Fermat varieties*, Proc. Kinosaki Symp. on Algebraic Geometry, Kyoto Univ., 2015, pp. 81–95.

Abstract: We answer a question of T. Shioda (posed in 1983) and show that, for any positive integer m prime to 6, the Picard group of the Fermat surface Φ_m is generated by the classes of lines contained in Φ_m. These results are extended to more general *Delsarte surfaces*, where the "obvious" divisors do not need to span the group, but the torsion is of a controlled size. We also suggest an algebraic restatement of the corresponding conjecture for Fermat varieties in higher dimensions.

36. A. Degtyarev, V. Florens, Ana G. Lecuona, *The signature of a splice*, Int. Math. Res. Not. IMRN **8** (2017), 2249–2283, doi:10.1093/imrn/rnw068.
37. A. Degtyarev, V. Florens, Ana G. Lecuona, *Slopes and signatures of links*, Fundamenta Mathematicae **258** (2022), 65–114, arXiv:1802.01836.
38. A. Degtyarev, V. Florens, Ana G. Lecuona, *Slopes of links and signature formulas*, Topology, geometry, and dynamics—V. A. Rokhlin-Memorial, Contemp. Math., vol. 772, Amer. Math. Soc., Providence, RI, 2021, pp. 93–105, arXiv:2002.02790.
39. A. Degtyarev, V. Florens, Ana G. Lecuona, *Slopes and concordance of links*, Algebraic & Geometric Topology (2022), arXiv:2202.04529 (to appear).

Abstract: We consider the signature of a colored link (a multivariate generalization of the classical univariate Levine–Tristram signature) in an integral homology sphere and study its behavior under the splice operation. We show that the signature is almost additive, with a correction term independent of the links. We interpret this correction term as the signature of a generalized Hopf link and give a simple closed formula to compute it. Applications are both old (Litherland's formula for satellite knots) and new (e.g., inductive computation of the signature of iterated torus links).

The formula suggested in [36] fails in one very special case. This case is treated in [37], leading to a new link invariant, called *slope*. Away from a certain singular locus, the slope is a rational function which can be regarded as a multivariate generalization of the Kojima–Yamasaki η-function. It is a certain ratio of two Conway potentials, *provided that the latter makes sense*; otherwise, it is a truly new invariant, which is confirmed by our experiments with the link tables. Using a similar construction for a special class of tangles, we also state generalized skein relations for the signature.

In [38], we summarize the results and announce the computation of the slope by means of C-complexes and its concordance invariance; a detailed proof of these facts is found in [39].

40. A. Degtyarev, I. Itenberg, A. S. Sertöz, *Lines on quartic surfaces*, Math. Ann. **368** (2017), no. 1, 753–809, doi:10.1007/s00208-016-1484-0.
41. A. Degtyarev, *Lines in supersingular quartics*, J. Math. Soc. Japan **74** (2022), 973–1019, doi:10.2969/jmsj/81998199.
42. A. Degtyarev, *Lines on smooth polarized $K3$-surfaces*, Discrete Comput. Geom. **62** (2019), no. 3, 601–648, doi:10.1007/s00454-018-0038-5.
43. A. Degtyarev, *Tritangents to smooth sextic curves*, Ann. Inst. Fourier **72** (2022), 2299–2338, arXiv:1909.05657.

44. A. Degtyarev, *Tritangents to smooth sextic curves* (full preliminary version), Preprint MPIM 19-51, 2019.
45. A. Degtyarev, S. Rams, *Counting lines with Vinberg's algorithm*, `arXiv:2104.04583` (to appear).
46. A. Degtyarev, S. Rams, *Lines on $K3$-quartics via triangular sets*, `arXiv:2301.04127` (to appear).

Abstract: We refine and reprove an old result by B. Segre (1943), showing that the maximal number of (real) lines in a (real) nonsingular spatial quartic surface is 64 (respectively, 56). We also give a complete projective classification of all quartics containing more than 52 lines: all such quartics are projectively rigid. (In particular, Schur's quartic discovered in 1882 is the only one with the maximal number 64 of lines.) As a consequence, a quartic defined over $\mathbb{Q}$ cannot have more than 52 lines defined over $\mathbb{Q}$. Any value not exceeding 52 can appear as the number of lines in an appropriate quartic; the other values are 54, 56, 60, and 64.

The second paper [41] treats supersingular quartics, where the number of lines is shown to be 40 or ≤ 32 if the characteristic of the field equals 2, and 112, 58, or ≤ 52 if the characteristic equals 3. Combining these results with geometric arguments, we also show that, in the case $\operatorname{char} \Bbbk = 2$, the sharp upper bound on the number of lines in a smooth quartic is 60.

In [42], [43], [44] we settle (in the affirmative) most conjectures suggested in [48] below and, more generally, establish sharp upper bounds on the number of (real) lines in a (real) smooth degree $2d$ model $X \to \mathbb{P}^{d+1}$ of a $K3$-surface X. A number of new examples of sextics in $\mathbb{P}^4$ and octics in $\mathbb{P}^5$ (as well as surfaces of higher degree) with many lines and sextics curves in $\mathbb{P}^2$ with many tritangents have been discovered.

Finally, in [45] we extend the approach developed in [42] to *singular* polarized $K3$-surfaces. We start from octics in $\mathbb{P}^5$ and show that, as in the smooth case, the sharp upper bound is 36 lines. Moreover, any octic with more than 32 lines is, in fact, smooth, whereas there are quite a few new examples of *singular* octics with 32 lines. A similar approach works for sextics in $\mathbb{P}^4$, resulting in at most 36 lines vs. 42 in the smooth case (a paper is currently in preparation). The most interesting case, namely that of spatial quartics, is handled in [46]. This case is much more involved, and we had to develop and treat the concept of the so-called *triangular sets*. Refining the results od D. Veniani, we prove that, in the presence of singularities, the sharp upper bound on the number of lines is 52, the only known example still being that of [48]. We also refine [40] and take the classification of configurations of lines in *smooth* quartics down to 49 lines.

47. A. Degtyarev, I. Itenberg, J. Ch. Ottem, *Planes in cubic 4-folds*, `arXiv:2105.13951` (to appear).

Abstract: We take the line counting problem to the next dimension and show that the number of 2-planes in a cubic 4-fold $X \subset \mathbb{P}^5$ takes values 405 (Fermat cubic only), 357, 351 (each realized by a single cubic), or at most 350. The last bound is not sharp: we expect more substantial gaps in the plane counts. The maximal number of real 2-planes in a real cubic 4-fold is 357.

48. A. Degtyarev, *Smooth models of singular $K3$-surfaces*, Rev. Mat. Iberoam. **35** (2019), no. 1, 125–172, `doi:10.4171/rmi/1051`.

Abstract: We show that the classical Fermat quartic has exactly three smooth spatial models. As a generalization, we give a classification of smooth spatial (as well as some other) models of singular $K3$-surfaces of small discriminant. As a by-product, we observe a correlation (up to a certain limit) between the discriminant of a singular $K3$-surface and the number of lines in its models. Thus, Schur's quartic (the only quartic containing the maximal number 64 of lines) can be characterized as the singular $K3$-surface of the smallest discriminant admitting a smooth spatial model. Similar bounds for smooth hyperelliptic $X \twoheadrightarrow \mathbb{P}^2$, sextic $X \hookrightarrow \mathbb{P}^4$, and octic $X \hookrightarrow \mathbb{P}^5$ models are obtained, and we conjecture that the number of lines in such models does not exceed 144, 42, and 36, respectively. (The former conjecture is still open, the two latter have been settled in [42].)

We also construct a $K3$-quartic surface with 52 lines *and singular points* (the best previously known example was a Delsarte surface with 40 lines, and the best known upper bound is 64) and a smooth quartic defined over $\mathbb{Q}$ with 46 lines defined over $\mathbb{Q}$ (the best known upper bound is 52, see [40], and no interesting examples were previously known).

49. A. Degtyarev, *Conics in sextic surfaces in* $\mathbb{P}^4$, Nagoya Math. J. **246** (2022), 273–304, `doi:10.1017/nmj.2021.3`.
50. A. Degtyarev, *800 conics in a smooth quartic surface*, Journal of Pure and Applied Algebra **226** (2022), 107077, `arXiv:2102.08163`.
51. A. Degtyarev, *Conics on Kummer quartics*, Tohoku Mathematical Journal, `arXiv:2108.11181` (to appear).
52. A. Degtyarev, *Conics on Barth–Bauer octics*, `arXiv:2210.06966` (to appear).

Abstract: This is yet another extension of the line counting problem: we bound the number of *conics* on polarized $K3$-surfaces. At present, the situation is far from satisfactory, even for quartics: the best known example has 432 conics (though, see [50]), whereas the best known upper bound is 5016. We modify the approach of [43], [44], [47] to tailor it to the conic counting problem and test it in the case of sextic surfaces in $\mathbb{P}^4$ (see [49]). We show that the maximal number of conics is 285 (a single surface) and the second maximal number is 261 (two surfaces, one maximizing the number of *real* conics). In all three extremal surfaces, all conics are irreducible. (*A priori*, we count both reducible and irreducible ones.)

In [50], [51] we start attacking the conic counting problem for spatial quartics: we construct an example of a quartic containing 800 irreducible conics and classify all configurations of conics in the Barth–Bauer family (which was the source of the previously known examples). It appears that 800 is indeed the sharp upper bound on the number of conics, both irreducible or reducible, in a smooth quartic surface. A similar classification for octics is done in [52], resulting in the conjectural bound of 176 conics. Considering $K3$-surfaces with a faithful action of a Mukai group, in [52] we also construct a double plane with 8910 conics; we conjecture that this is the maximum.

53. E. Brugallé, A. Degtyarev, I. Itenberg, F. Mangolte, *Real algebraic curves with large finite number of real points*, European J. Math. **5** (2019), no. 3, 686–711, `doi:10.1007/s40879-019-00324-9`.

Abstract: We address the problem of the maximal finite number of real points of a real algebraic curve (of a given degree and, sometimes, genus) in the projective plane. We improve the known upper and lower bounds and construct close to optimal curves of small degree. Our upper bound is sharp if the genus is small as compared to the degree. Some of the results are extended to other real algebraic surfaces, most notably ruled.

54. A. Degtyarev, I. Itenberg, *Real plane sextics without real points* (to appear).

Abstract: We prove that the equisingular deformation type of a simple real plane sextic curve with smooth real part is determined by its real homological type, i.e., the polarization, exceptional divisors, and real structure recorded in the homology of the covering $K3$-surface. As an illustration, we obtain an equisingular deformation classification of real plane sextics with empty real part (for completeness, we consider the few non-simple ones as well).