Math 310 Topology, Spring 1999
Final Exam

You can use your textbooks and/or lecture notes

Due on 5/11/99

Name: ______________________________
I.D.: ______________________________

Problem 1.

(1) Show that a connected normal space with more than one point is uncountable.
(2) Show that a connected regular space with more than one point is uncountable.

Problem 2.

(1) Show that any locally compact second countable Hausdorff space is metrizable.
(2) Is any second countable metrizable space locally compact?

Problem 3. Show that the closed ray $X = [0, \infty)$ does not admit a two-point compactification. More precisely, there is no compact Hausdorff space Y containing X as a dense subset and such that $Y \setminus X$ consists of two points. (*Hint*: show that the two points constituting $Y \setminus X$ cannot be separated.)

Problem 4. Show that the trefoil knot bounds a surface of genus 1, i.e., there is a way to embed a torus with hole into $\mathbb{R}^3$ so that it's boundary is the trefoil knot.

Problem 5. Show that any **triangulation** of a torus consists of at least 7 triangles. (*Hint*: Use Euler characteristic.)