Math 310 Topology, Spring 1998
Final Exam

You can use your textbooks and/or lecture notes

Name: ______________________________
I.D.: ______________________________

Problem 1 (30 pts).

(1) Show that any locally compact second countable Hausdorff space is metrizable.
(2) Is any second countable metrizable space locally compact?

Problem 2 (25 pts). Let X be a completely regular topological space, $A, B \subset X$ disjoint closed subsets, and A compact. Show that there is a continuous function $f\colon X \to [0,1]$ such that $f|_A \equiv 0$ and $f|_B \equiv 1$.

Problem 3 (25 pts). Show that $\mathbb{R}^n$, $n > 2$, is not homeomorphic to $\mathbb{R}^2$. (*Hint*: remove a point.)

Problem 4 (40 pts). Show that the closed ray $X = [0, \infty)$ does not admit a two-point compactification. More precisely, there is no compact Hausdorff space Y containing X as a dense subset and such that $Y \smallsetminus X$ consists of two points. (*Hint*: show that the two points constituting $Y \smallsetminus X$ cannot be separated.)

Problem 5 (30 pts). Compute the fundamental groups of the following spaces:

(1) the n-dimensional torus $T^n = (S^1)^n$;
(2) the solid torus $D^2 \times S^1$;
(3) the annulus $\{z \in \mathbb{C} \mid 1 < |z| < 2\}$;
(4) the Möbius band;
(5) the space $\mathbb{R}^4 \smallsetminus \mathbb{R}^2 = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1^2 + x_2^2 \neq 0\}$;
(6) the letter 'R'.