Math 310 Topology, Spring 1998
Midterm 1

You can use your textbooks and/or lecture notes

Name: ______________________________
I.D.: ______________________________

Problem 1. A topological space X is said to be *completely normal* if every subspace of X is normal. Show that X is completely normal if and only if for every pair A, B of *separated* subsets of X (i.e., subsets such that $\operatorname{Cl} A \cap B = A \cap \operatorname{Cl} B = \varnothing$) there are disjoint open sets containing them. (*Hint*: if X is completely normal, consider $X \smallsetminus (\operatorname{Cl} A \cap \operatorname{Cl} B)$.)

Problem 2. A subspace A of a topological space X is called a *retract* of X if there is a continuous map $\rho\colon X \to A$ such that $\rho(a) = a$ for any $a \in A$.

(1) Show that a retract of a Hausdorff space is closed.
(2) Show that the unit circle S^1 is a retract of $\mathbb{R}^2 \smallsetminus \{0\}$.

Problem 3. Enumerate all the nonhomeomorphic topologies on a three point set.

Problem 4. Let X be a 2 nd countable topological space and $A \subset X$ an uncountable subspace.

(1) Show that A is not discrete.
(2) Show that A contains at least one of its limit points.
(3) Show that A contains uncountably many of its limit points.