Date: November 28, 2006

Time: 8:40–10:30 am

NAME: _____

STUDENT ID: _____

DEPARTMENT: CS   EE   IE

Math 220-01, Fall 2006

MIDTERM II

VERY IMPORTANT

1. Indicate clearly and unambiguously your final result. In proofs, state explicitly each claim.

2. Do not misread the questions or skip parts thereof. If you did, do not complain.

3. If you believe that a problem is misstated, do not solve it; explain your point of view instead.

4. Each problem has a reasonably short solution. If your calculation goes out of hands, something must be wrong.

TERMS AND CONDITIONS

1. This exam consists of 5 questions of equal weight.

2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.

3. Show all your work. Correct answers without sufficient explanation might <u>not</u> get full credit.

4. Calculators are <u>not</u> allowed.

Please do <u>not</u> write anything below this line.

1	2	3	4	5	TOTAL

1. In $\mathbb{R}^3$ with the standard inner product find the distance between the vector $\mathbf{u}$ and the plane spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$, where

$$\mathbf{u} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \qquad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

2. The inner product on $\mathbf{R}^4$ is given by $(\mathbf{a}, \mathbf{b}) = a_1 b_1 + 2a_2 b_2 + a_3 b_3 + 2a_4 b_4$. Use the Gram-Schmidt process to find an orthonormal basis in $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where

$$
\mathbf{u}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \\ 0 \end{bmatrix}, \quad
\mathbf{u}_2 = \begin{bmatrix} 5 \\ -1 \\ 4 \\ 3 \end{bmatrix}, \quad
\mathbf{u}_3 = \begin{bmatrix} 10 \\ -5 \\ 5 \\ 11 \end{bmatrix}.
$$

3. For the matrix

$$A = \begin{bmatrix} -2 & 2 & 3 & 7 & 1 \\ -2 & 2 & 4 & 8 & 0 \\ -3 & 3 & 2 & 8 & 4 \\ 4 & -2 & 1 & 5 & -7 \end{bmatrix}$$

find the rank, nullity, and bases for the row, column, and null spaces.

4. The inner product on the space P_2 of polynomials of degree ≤ 2 is given by

$$(p, q) = \int_{-1}^{1} t^2 p(t) q(t) \, dt.$$

Let $p(t) = (t-1)^2$ and $q(t) = (t+1)^2$. Find the norms $\|p\|$ and $\|q\|$, the angle between p and q, and the distance between p and q.

5. Let
$$C = \begin{bmatrix} a & 1 \\ 1 & 5 \end{bmatrix}.$$
Find all values of a for which the function $(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T C \mathbf{y}$ is an inner product on $\mathbf{R}^2$.