Date: October 31, 2006
Time: 8:40–10:30 am

NAME: ______________________________

STUDENT ID: ______________________________

MATH 220-01 MIDTERM I

IMPORTANT

1. This exam consists of 5 questions of equal weight.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.

Please do not write anything below this line.

1	2	3	4	5	TOTAL

1. Solve the system and find a basis for the solution space of the corresponding homogeneous system:

$$\begin{array}{rcrcrcrcrcr} 2x_1 & - & x_2 & + & x_3 & - & x_4 & + & 3x_5 & = & 0 \\ 4x_1 & - & 2x_2 & + & 2x_3 & - & 2x_4 & + & 5x_5 & = & 1 \\ 4x_1 & - & 2x_2 & + & 4x_3 & & & + & 13x_5 & = & -3 \\ -6x_1 & + & 3x_2 & + & x_3 & + & 7x_4 & + & 2x_5 & = & -3 \end{array}$$

2. Let A and B be two $(n \times n)$-matrices, and assume that B is singular. Prove that AB is also singular.

3. If exists, find A^{-1} for

$$A = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}.$$

4. Consider the polynomials $\mathbf{v}_1 = 3t^3 + 2t^2 - t + 2$, $\mathbf{v}_2 = 4t^3 + 2$, $\mathbf{v}_3 = 3t^3 + 2t^2 - t + 2$, $\mathbf{v}_4 = 5t^3 + 6t^2 - 3t + 2$, $\mathbf{v}_5 = 4t^2 - 2t + 1$ as vectors in the space P_3 of polynomials of degree up to 3.

1. Does the vector $\mathbf{u} = 1$ belong to the subspace $W = \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$?

2. Are $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$ linearly independent?

3. Do $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$ span P_3?

4. Find a basis and the dimension of the subspace $W = \mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$.

5. Find the coordinates of the vector $\mathbf{v} \in \mathbf{R}^3$ with respect to the basis S, where

$$\mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix} \quad \text{and} \quad S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}.$$