Date: December 9, 2004
Time: 10:40–12:30

NAME: _____

STUDENT ID: _____

DEPARTMENT: CS EE IE

**Math 220-01, Fall 2004**

**MIDTERM II**

**VERY IMPORTANT**

**1.** Indicate clearly and unambiguously your final result. In proofs, state explicitly each claim.
**2.** Do not misread the questions or skip parts thereof. If you did, do not complain.
**3.** If you believe that a problem is misstated, do not solve it; explain your point of view instead.
**4.** Each problem has a reasonably short solution. If your calculation goes out of hands, something must be wrong.

**TERMS AND CONDITIONS**

**1.** This exam consists of 5 questions of equal weight.
**2.** Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
**3.** Show all your work. Correct answers without sufficient explanation might not get full credit.
**4.** Calculators are not allowed.

Please do not write anything below this line.

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| 1 | 2 | 3 | 4 | 5 | TOTAL |
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| | | | | | |

**1.** Let $P_4$ be the space of polynomials of degree $\leq 4$. Prove that

$$(p, q) = \int_0^2 (t-1)^2 p(t)q(t)\, dt$$

is an inner product and, given this inner product, find the kernel of the linear transformation $\mathrm{pr}_W \colon P^4 \to W$, where $W = P^2 \subset P^4$.

**2.** The inner product on $\mathbb{R}^4$ is given by $(a, b) = 2a_1b_1 + a_2b_2 + a_3b_3 + 2a_4b_4$. Use the Gram-Schmidt process to find an orthonormal basis in $W = \text{Span}\{u_1, u_2, u_3\}$, where

$$u_1 = \begin{bmatrix} -2 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \qquad u_2 = \begin{bmatrix} -1 \\ 4 \\ 5 \\ 3 \end{bmatrix}, \qquad u_3 = \begin{bmatrix} -5 \\ 5 \\ 10 \\ 11 \end{bmatrix}.$$

**3.** *Trace* $\operatorname{Tr} C$ of a square $(n \times n)$-matrix $C = [c_{ij}]$ is the sum $\sum_{i=1}^{n} c_{ii}$ of its diagonal elements. Prove that for any two $(m \times n)$-matrices $A$ and $B$ one has $\operatorname{Tr}(A^{\mathrm{T}}B) = \operatorname{Tr}(B^{\mathrm{T}}A)$.

**4.** Find the polynomial $p(t)$ of degree $\leq 3$ such that $p(-1) = p(1) = 0$ and the value of

$$\int_{-1}^{1} |p(t) + 3t - 5|^2 dt$$

is minimal possible.

**5.** Let

$$C = \begin{bmatrix} 3 & a \\ a & 2 \end{bmatrix}.$$

Find the values of $a$ for which the function $(x, y) = x^{\mathrm{T}} C y$ is an inner product on $\mathbb{R}^2$.