Date: November 7, 2002

Time: 1:40–3:30 pm

NAME: _____

STUDENT ID: _____

MATH 220-05 MIDTERM I

IMPORTANT

1. This exam consists of 5 questions of equal weight.

2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.

3. Show all your work. Correct answers without sufficient explanation might <u>not</u> get full credit.

4. Calculators are <u>not</u> allowed.

Please do <u>not</u> write anything below this line.

1	2	3	4	5	TOTAL

1. Solve the system and find a basis for the solution space of the corresponding homogeneous system:

$$
\begin{aligned}
2x_1 &- x_2 + x_3 + 2x_4 + 3x_5 &= 2 \\
6x_1 &- 3x_2 + 2x_3 + 4x_4 + 5x_5 &= 3 \\
6x_1 &- 3x_2 + 4x_3 + 8x_4 + 13x_5 &= 9 \\
4x_1 &- 2x_2 + x_3 + x_4 + 2x_5 &= 1
\end{aligned}
$$

2. Let A and B be, respectively, an $(m \times q)$-matrix and a $(q \times n)$-matrix. Prove that

$$\operatorname{rank}(AB) \leq \min\{\operatorname{rank} A, \operatorname{rank} B\}.$$

3. If exists, find A^{-1} for

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 5 & 9 & 2 & 6 \end{bmatrix}.$$

4. Find the dimension and a basis for Span S, where

$$S = \left\{ \begin{bmatrix} 3 \\ 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -2 \\ -1 \end{bmatrix} \right\}.$$

5. Find a basis for the space $V \subset P_5$ of polynomials p of degree up to 5 such that

$$p(-2) = \frac{dp}{dt}(-2) = \frac{d^2p}{dt^2}(-2) - \frac{d^3p}{dt^3}(-2) = 0.$$

What is the dimension of this space?