Date: December 15, 1997
Time: 1:40-3:30

NAME:..

STUDENT NO:..

DEPARTMENT: CS EE IE

MATH 220 FINAL EXAM

IMPORTANT

1. This exam consists of 6 questions of equal weight.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.

Please do not write anything below this line.

1	2	3	4	5	6	TOTAL
20	20	20	20	20	20	120

1. If possible, diagonalize the matrix and find a basis in which it has diagonal form:

$$A = \begin{bmatrix} -2 & 0 & -1 & -1 \\ -3 & 1 & -3 & -3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -1 & -3 \end{bmatrix}.$$

2. Find A^{-1} for

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

3. Let M_n be the space of $(n \times n)$-matrices, $B \in M_n$ a fixed matrix, and $L\colon M_n \to M_n$ the operator given by $L(A) = BA$. Prove that λ is an eigenvalue of L if and only if $\det(\lambda I_n - B) = 0$.

4. Find an orthogonal basis (do <u>not</u> normalize) for the space P_3 of polynomials of degree at most 3 with the inner product

$$(p, q) = \int_0^2 p(t) \cdot q(t)\, dt.$$

(*Hint*: the amount of calculation reduces substantially if you choose an appropriate basis to start with.)

5. Define the *trace* of a square $(n \times n)$-matrix $A = [a_{ij}]_{1 \le i,j \le n}$ to be $\operatorname{trace} A = \sum_{i=1}^{n} a_{ii}$. Show that the traces of similar matrices coincide.

6. Let

$$v_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -2 \\ 4 \\ 0 \\ 2 \\ -2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -3 \\ 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 2 \\ -4 \\ -1 \\ -3 \\ 4 \end{bmatrix}$$

be vectors in $\mathbb{R}^5$ with the standard inner product and $W = \operatorname{Span}\{v_1, v_2, v_3, v_4\}$. Find a basis for $W^\perp$. What are $\dim W$ and $\dim W^\perp$?